Fractal Generating Techniques

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Abstract: Globally, researchers have been trying to produce the most comprehensive description of introducing fractals and utilizing them in different sciences. Their effort lies in elaborating and recruiting the scope of fractals in science and applications, such as antenna design, computer software programming. Further, the modern world needs the products to be compact, efficient and economically suitable, where fractals bring such recommendations. In this review article, we present a briefed description of fractals, types of fractals, merits of fractals and, most importantly, fractal generator techniques along with their mathematical techniques and software.

1 INTRODUCTION

In mathematics, a fractal is a complete, iterative-like, and self-alike mathematical set whose Hausdorff aspect or direction firmly overrides its topological dimension. Fractals are available universally in nature because of their predisposition to seem approximately identical at different aspects, as appears in the consecutively trivial amplifications of the Mandelbrot set. Further, fractals have alike arrangements at progressively small sizes that is similarly known as intensifying symmetry or unfolding symmetry (Mandelbrot, 1983). When this repetition is strictly the same as the relating generated almost same copy at every scale, like what appears in the Menger sponge, a fractal has a self-similar arrangement.

Fractal is an, somehow, irregular or disjointed geometric shape, which can be sub-partitioned in parts, where each part is roughly a smaller copy of a whole fractal object (Paul, 1991). It is a natural phenomenon or a mathematical expression, which has a repeating pattern that displays at every scale. If the replication is the same at every scale, it is called a self-similar pattern. Fractals can also be nearly the same at different levels and includes the idea of a detailed pattern that repeats itself. As seen in nature, most physical systems, structures, objects and works are not east-to-recognize systematic geometric and are not mathematically calculated shapes of the standard geometry.

Many patterns of fractals can be generated by utilizing inspirations from the areas of natural sciences. An example of such an inspiration is the diffusion-limited aggregation (DLA) that describes, apart from other descriptions, the diffusion-aggregation of zinc ions in an electrolytic solution on electrodes. Other examples of naturally generated fractals because of their ultimate structure are the flowers, vegetables, etc. Fractal would appear when analyzing ice particles; hence, it shows a dramatic presentation of fractal growths as monitored by utilizing a specialized telescopic tool. Further, fractal shows itself in the structure of many living organs and bodies of animals. For instance, fractal patterns have a critical role in fortifying and shaping the shell in snails, where their shells revolve in an obvious way of fractal shape. Such observation is noticeable in almost every aspect of life (Douglas et al., 2003). In addition, Large-scale objects like galaxies and small-scale items like atoms are all offering different forms

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of fractal generating initiatives. Moreover, movements, motion, and interpolation mathematical processes in science exhibit stochastic models that contain fractal behavior; hence, every single item in the universe can initiate a fractal behavior at some point.

This reviewing work is mostly dedicated for the techniques by which fractals are generated, because fractals have templates and/or functions with a prototype item. At that point, the prototype grows within some patterns to build the whole structure of the fractal body. The paper, however, has been divided into many divisions to comprehend and visualize a full panorama of what to discuss about fractal initiation and how to generate fractals. The first division talks about an explanation of where the idea of fractals have come from. In the next division, merits of utilizing fractals are discussed, whereas the following section collects many fractal generation techniques with some supporting explanation. Moreover in this work, the next section depicts the mathematical representation of fractal generation. Before the conclusion and references, the final section mentions the computer software programs dedicated for fractals.

2 FRACTALS CONCEPT

There have been tremendous number of researchers and mathematicians around the globe trying to elaborate the key philosophy of the fractal spreading in the universe. The apparent behavior of natural fractal pattern expose an attitude of the nature to build living and non-living objects that mimic their own items; i.e. as if the nature's printer types the same generator with different scaling factors to have such infinite-like patterns; hence, nature would select the simplest procedure to fill space and size. At that point, the simplest way would be to accumulate almost-similar structures in, somehow, different styles to expose the final product.

Artificial fractals are useful to express and modulate objects that may contain a base to start from, such as decorations, computer images, civil structures, architecture, interpolation, engineering tools, economics, etc. The most important merit of artificial fractals is the ability to comprehend current solution needs and to create the suitable fractal texture that fits the different desires. On the other hand, natural fractals follow the needs of nature, without an exact explanation as mentioned earlier. However, it still needs further discovery to explain the origin of fractals and why/how it has reached such forms, but it is probably not able to explain without utilizing deeper physical, mathematical theories and maybe super computers and algorithms.

3 MERITS OF USING FRACTALS

Fractals have grabbed many properties into account, inspired primarily from nature. Depending on the field that uses iterative geometric properties, merits of fractals can be classified into some items as listed (Douglas et al., 2003).

1. In-fit size structure.
2. Low profile packages
3. Conformal
4. Broadband and/or multiband
5. Fast growing attitude
6. Predictable approach throughput
7. Easiness of programming and modelling
8. Fashion style and artistic design
9. Forecast of many life representations
10. Key to explain prospective and existing theorems.

According to the aforementioned and other properties, fractals and similar geometric designs have become the desirable figures with respect to researchers, designers, and programmers.

4 AGGREGATION OF SOME FRACTAL GENERATION FIGURES

Over ages, mathematicians and scientists have found and developed fractal shapes depending on the application they intend to adopt. This section presents a comprehensive overview of some common fractal geometries that have been developed or discovered. These designs have been used in developing modern and innovative designs of technological and engineering system structures, such as demographic mapping, computer software, systems models, microwave assemblies and antennas.

4.1 Sierpinski Gasket

The first iterations in the construction of the Sierpinski gasket are shown in Figure 1. The process of the geometry of such construction is a fractal beginning with an equilateral triangle, as illustrated in the first stage of Figure 1. The next iteration in the construction is to remove a central triangle that is located at the mid of the original triangle. This newly removed triangle has vertices at the centers of each side of the original triangle as shown in Stage 1. This process repeats itself for the remaining three triangles, as shown in Stage 2, 3, and 4 for the same figure. Consequently, the Sierpinski-gasket fractal is generated by carrying out this consecutive process an infinite amount of times. Further, Sierpinski gasket is an example of a generally self-similar fractal. From an RF engineering viewpoint, a practical clarification of Figure (1) is that the black triangular areas characterize a metallic conductor; while the white triangular areas characterize regions where metal has been removed (Douglas et al., 2003).

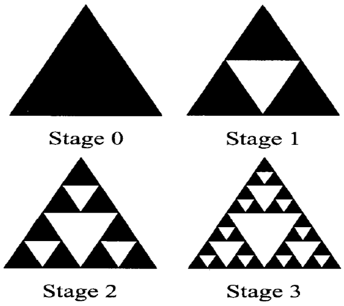

Figure 1: Sierpinski-gasket fractal construction.

4.2 Fracal by Entrenches

In this design, a new idea of patterning a fractal antenna comes in a reality, where a circular patch that contains entrenches was utilized in this process, as shown in Figure 2. The process of this design takes the manner of internally enclosed circles of entrenches. Further, the first far tire has a designated number of slots, where other tires have the same number as well with smaller size of slots in each circle.

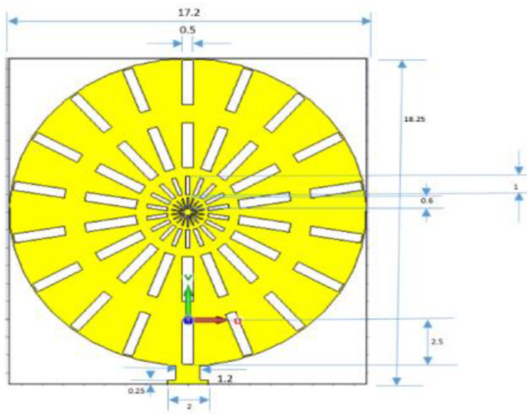

Figure 2: Fractal of entrenches.

4.3 Koch Snowflake Fracta

This design is another pattern of fractals, and is well mentioned by many researchers around the world. For instance, it takes its shape primarily from the microscopic-scale design of a snowflake unit.

The design starts out as a solid equilateral triangle like the Sierpinski gasket, as illustrated in Figure 2. Nevertheless, unlike the Sierpinski gasket that is formed by downgrading the size of the triangles from the original structure, the Koch snowflake is accumulated by adding downgrading triangles into the structure in an iterative style, as in Figure 3 (Douglas et al., 2003).

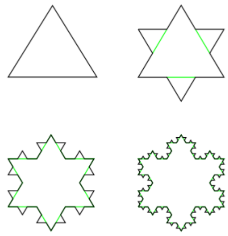

Figure 3: Koch snowflake fractal iterations.

4.4 Self-similarity Fractals

It refers to objects that contain smaller copies or duplicates of itself at arbitrary scales. Figure 4 shows an example of a natural self-similarity fractal. Such fractal can further be divided into three items of self-similarity fractals.

4.4.1 Exact Self-similarity Fractal

The fractal is the same at diverse balances. Such fractal is the sturdiest kind of the self-similarity type.

4.4.2 Quasi-self-similarity Fractal

The fractal is about to be alike at diverse balances. This one is a fewer specific system of self-similarity type. Such fractals comprehend minor duplicates of the whole fractal in slanted forms.

4.4.3 Statistical Self-similarity

It is the weakest type of self-similarity; hence, this fractal has computational or statistical measurements that are preserved across scales. However, most famous definitions of fractals imply some meaning of statistical self-similarity (a dimension of a fractal is a numerical measurement that is kept across scales). Further, random fractals are kind of fractals that are computationally or statistically self-similar, but neither quasi-self-similar nor exactly self-similar (Nicoletta, 2013).

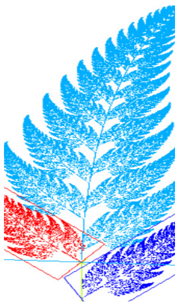

Figure 4: Barnsley fern exhibiting self-similarity fractal behavior.

4.5 Pixel-covering Method Fractal

It is useful to compute the fractal dimensions of objects, such as leaves, based on many plant species acquired from several places for the sake of plant classification and identification. Therefore, both contour fractal dimension and the contour & nervure fractal dimension can distinguish leaves between different types effectively despite a little deficiencies. The process works by adding the fractal dimension of nervure details into the whole classification system that can determine leaves more robustly than that of contour and contour & nervure. Figure 5 depicts a classification process by using pixel-covering method (Wei et al., 2009).

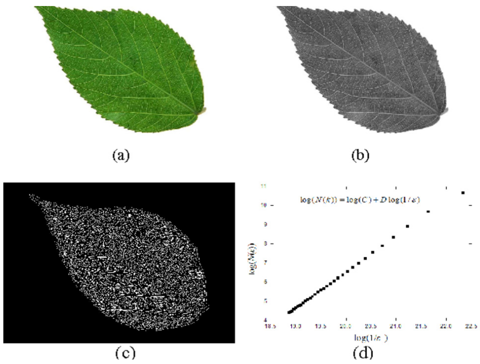

Figure 5: A classification process by pixel-covering method.

4.6 Fractals for Geo-chemical Exploration Data of a Geological Area

In this type of fractals, multi-fractal method is carried out to process up to 1:200000 stream sediment geo-chemical examination data of a geographical area. Fractal dimension characteristics of a number of elements connected with minerals are gained based on (C-A) fractal technique, in addition to diverse geo-chemical anomaly stages and components mixtures (Shili Liao et al., 2012).

4.7 Growing-to-the-inside Fractals

This technique in generating fractals was proposed by some researchers. Abolfazl Azari proposed an example of such design (Abolfazl, 2011). The design takes the shape of octagonal arrays formed by placing elements in an equilateral triangular net. Hence, these arrays can be viewed as involving of a single item at the center, bounded by several concentric eight element circular arrays. Figure 6 depicts the iterations of the proposed design of such fractal.

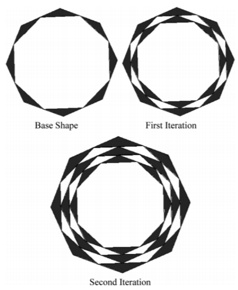

Figure 6: The iterations of the proposed fractal by (Abolfazl, 2011).

4.8 Space Filling Techniques as Fractals

Space filling techniques can serve as a method of generating fractals. They take the merit of packing extra lines and curves as the rank of the technique gets higher. For instance, the most famous techniques to fill spaces are Hilbert, Peano, Moore, Dragon, Gosper, Koch techniques. Figure 7 shows three patterns that fill spaces in a deterministic mathematical style.

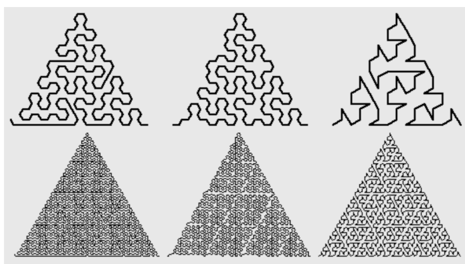

Figure 7: Three curves for filling spaces as fractals (teachout).

4.9 Three-dimensional Fractals

They exist in nature and mathematics and cover many aspects of fractalism in nature and artificial computations. Most natural fractals are, somehow, in the form of 3D pattern; hence, they mostly change their way of spreading in more than one plane. Moreover, some examples of three-dimensional fractals are DNA, neurons, natural or artificial surfaces, soil, clouds, etc. Figure 8 exposes an example of a natural three-dimensional fractal.

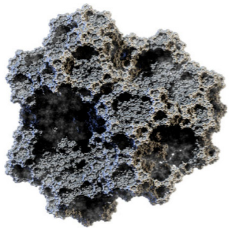

Figure 8: A 3-D fractal shape.

4.10 Fractals of Multi-scroll Chaotic Attractors

Attractors are sets of numerical values of a system that goes to evolve, though they sometimes look complicated and random. In fractals, sets of multi-scroll chaotic attractors are hard to simulate and to be put in a mathematical model to represent the fractal structure. However, Lu Chen attractor and the modified Chua chaotic attractor are examples of modeling attractors and are applicable to comprehend the fractal implementation. Figure 9 shows an example of a multi-scroll chaotic attractor fractal.

Figure 9: Fractal of multi-scroll chaotic attractors.

5 MATHEMATICAL REPRESENTATION OF FRACTAL INITIATION

Fractals have wide scopes of mathematical characterization that fill a specific boundary of occupation. Further, large number of fractals are deterministic, i.e. they commonly can be predicted by utilizing mathematical and logical formulas. In addition, it is likely to have fractals that are hard or impossible to obtain a computational formula of representation. In such a case many naturally built fractal shapes, such as coral reefs, trees, landscapes, etc. As indicated by many researchers around the world, it is not completely known the precise reason that explains the mathematical patterning of fractals in nature. The following lists some mathematical patterns of fractal characterization.

5.1 Fractal Interpolation

Chih-Chin Huang, Shu-Chen Cheng, and Yueh Min Huang in the refrence (Chih-Chin et al., 2010) investigated a new algorithm to generate a new interpolation scheme. Such an algorithm is helpful in the techniques concerning generating and forecasting fractal numbers out of a few numbers. Figure 10 shows an example of fractal interpolation of images.

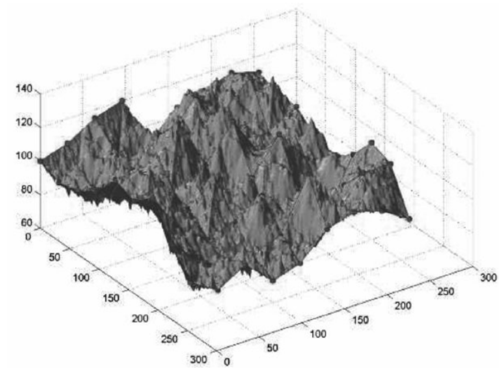

Figure 10: Fractal interpolation of images (Pantelis et al., 2007).

5.2 Iterated Function Systems

Iterated function systems, or (IFS), represent a very various technique for properly generating a wide-ranging useful fractal structures. Such iterated function systems stand on the application of a series of affine transformations as in figure 11 (Douglas et al., 2003).

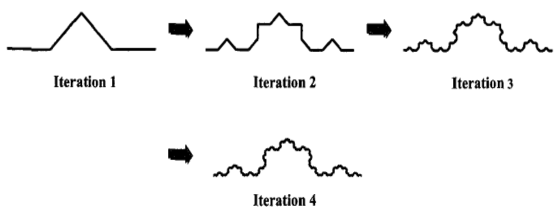

Figure 11: the construction of the standard Koch curve via an iterated function system (IFS) approach.

5.3 Circulation of Fractals

As an example of such technique is what appears earlier in this article in the item [Fractal by entrenches]. When the designer used multi-circles of entrenches (18 circle per rotation) that have the same characteristics with different scales.

5.4 Super-formula

Johan Gielies presented this formula. Such a formula mostly describes the natural fractal phenomenon (Nicoletta, 2013). The following equation describes the general formula of this theory.

$$r = f(\emptyset)\frac{1}{\sqrt[n_1]{\left(\left|\frac{1}{a}\cos\left(\frac{m}{4}\emptyset\right)\right|\right)^{n_2}+\left(\left|\frac{1}{a}\cos\left(\frac{m}{4}\emptyset\right)\right|\right)^{n_3}}}$$

5.5 Logarithmic Fractals

This technique is mostly useful in the fractals that relate to natural organs. Figure 5 represents a vital organ having such technique (Wei et al., 2009).

5.6 Pseudo Random Key-stream Generator

Pseudo random number generators have played a critical research point due to the demand on quality-encoded content that is essential in all of the structure of the communication networks. Such technique has many examples and can be found in variety of research papers as in figure 12.

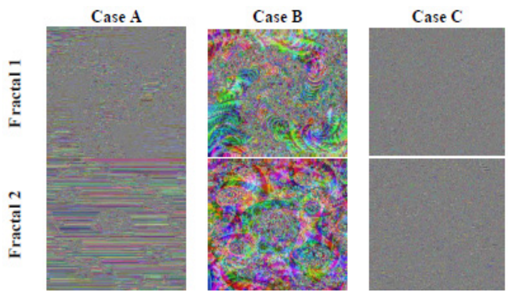

Figure 12: Example of Pseudo Random Keystream Generator using Fractals (Sherif et al., 2013).

5.7 Space filling Curvature Formulas

There are many formulas to represent such curvatures, such as Cantor function, Tietze extension theorem, Euclidean metric, Lindenmayer system, L-System, segment division, Weierstrass function, other deterministic and un-deterministic methods. Figure 13 shows an example of Cantor function that has a fractal extension in its higher order formulas.

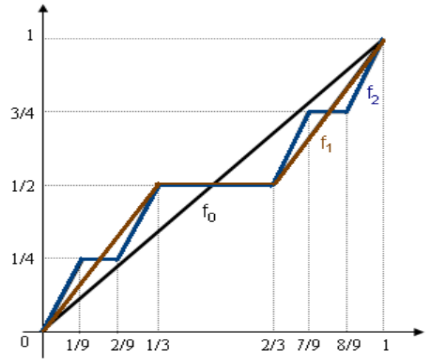

Figure 13: A graph of Iterative Construction of Cantor function.

5.8 NP Generator Model

It includes iterating a Narrow Pulse within a specific shape as in figure 14.

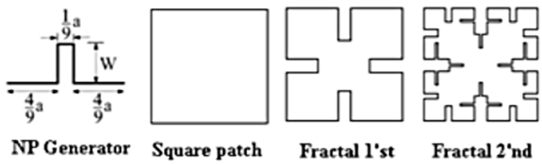

Figure 14: A model of NP generator for a square patch (Mahatthanajatuphat et al., 2007).

5.9 Triangular Sub-divisions

This kind of fractal formation may exist in fractal-related computer processors and arrays (Wainer, 1988). An example of such generator can be shown in figure 15.

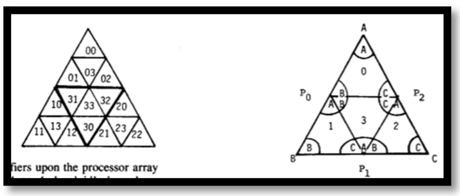

Figure 15: A sub-divisions method process (Wainer, 1988).

5.10 Multi-scroll Chaotic Attractor Generator

Many models represent such structures and have an evolving approach, resembling Lu Chen and the modified Chua chaotic attractors. Figure 16 shows an example of a system of fractal processes and transformation Φ (Bouallegue, 2011).

$$
\Phi \begin{cases} (x_0, y_0) \\ (x_{i+1}, y_{i+1}) = P_1(\alpha x_i + \gamma, \beta y_i + \lambda) \\ (x_{i+2}, y_{i+2}) = P_2(x_{i+1}, y_{k+1}) \\ (x_{i+3}, y_{i+3}) = T_1(x_{i+2}, y_{i+2}) \\ (x_{i+4}, y_{i+4}) = P_3(x_{i+3}, y_{i+3}) \\ \vdots \\ (x_{j+1}, y_{j+1}) = T_k(x_{j-1}, y_{j-1}) \\ \vdots \\ (x_m, y_m) = P_{m-k}(x_{m-1}, y_{m-1}) \end{cases}
$$

Figure 16: System of fractal processes and transformation as an example of Multi-scroll chaotic attractor generator.

5.11 Random Iteration Algorithm

The preliminary set is a single point and at each point of iteration, only one of the essential affine transformations is used to compute the following level (Ankit et al., 2014). Moreover, Hsuan T. Chang presented a group of decoded images by the random iteration algorithm as in figure 17 (Hsuan, 2001).

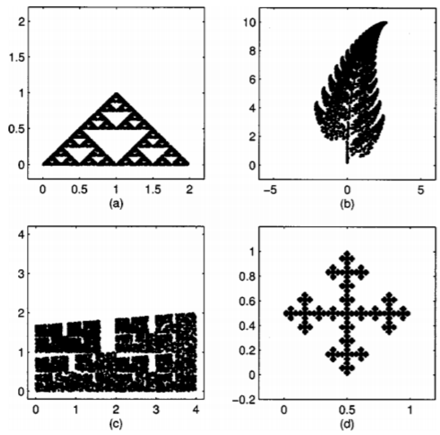

Figure 17: Decoded images of fractals by the random iteration algorithm: (a) Sierpin´ski triangle, (b) fern, (c) castle, and (d) snowflake.

5.12 Stochastic Fractal Search (SFS) Algorithm

Such algorithm is considered to be a development of Evolutionary Algorithms (EAs), and it uses the diffusion merit that is seen frequently in random fractals. The particles in such algorithm discover the searching space more powerfully and is used optimization processes (Salimi, 2015).

6 FRACTAL-GENERATING SOFTWARE

Fractal generation software represents every kind of software of graphics generating images of fractals. However, there are numerous programs of fractal generation obtainable, together open and profitable. Mobile applications are accessible to play with fractal designs. Various programmers generate fractal software for their interest due to the innovation and due to the challenges in comprehending the related mathematical problems. Therefore, generating fractals has directed many large difficulties and projects for pure mathematics.

Mainly, there are two key approaches of two-dimensional generation of fractals. First of which is to conduct a process of iteration to simplify calculations by recursion generation (Daniel, 2017). On the other hand, the other chief approach is with Iterated-Function-Systems (IFS) that consist of an amount of affine alterations. In method number one, every pixel within fractal images is assessed based on a function and, at that time, colored, beforehand the similar procedure is conducted to the following pixel. Hence, the previous technique characterizes the traditional stochastic method, whereas the second builds a linear model of fractals. Utilizing recursion have permitted program operators to generate complicated images over modest direction [19-21].

- **Chaotica:** A commercialized fractal art software and renderer prolonging flam3 as well as Apophysis function.
- **Apophysis:** An open-source fractal flame software intended for Microsoft Windows and Macintosh.
- **Fractint:** Free software to display numerous types of fractals. The software was created on MS-DOS, after that transported to the Atari ST, Macintosh and Linux.
- **Electric Sheep:** An open-source spread screen saving software, and was established by S. Draves.
- **Kalles Fraktaler:** A free Windows built fractal zooming program.
- **Milkdrop:** A hardware with accelerated music visualizing plugin intended for Winamp that was initially advanced by R. Geiss.
- **XaoS:** A fractal zooming software with interaction.
- **Fyre:** An open source cross-platform apparatus intended for creating images centered about histograms of repeated chaotic functions.

- **OpenPlaG:** It generates fractal by sketching modest functions and is PHP based.
- **MojoWorld Generator:** It was a commercial fractal landscape initiator intended for Windows.
- **Sterling:** Freeware fractal generator software inscribed with C language.
- **Picogen:** A freeware open source cross platform terrain initiator written in C++.
- **Terragen:** A generator of fractal terrain, which can handle animations in Mac OS X and Windows.
- **Wolfram Mathematica:** Dedicated for many computer science software and for creating fractal images as well.
- **Ultra Fractal:** A rendering generator for fractals in Mac OS X and Windows.

7 CONCLUSION

In this research-reviewing article, the authors give a summarized idea of collecting what many researchers have been investigating in the field of fractals. During decades, the concept of understanding fractals in nature has led to employ this idea in the industrialized and artificial forms. Backed from its history, area of fractals is developing in terms of classification, benefits, future employment, further understanding and relation with life origins. It is expected, in addition, that such field in researching fractals and utilizing it in modern-life employment would enhance the efforts in finding new algorithms to expand the atmosphere that fractals deploy. Furthermore, there is a fact that utilizing fractals in the fields of microwave and antenna engineering has practically occupied the most complicated and expanded effort by engineers and researchers through theory, simulation, and prototyping.

For the sake of future direction, fractal generating techniques will develop further to comprehend the increasing demands in variety of applications; hence, the incoming trend is seeking for more compact and practical designs and concepts.

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