Constructive Solid Geometry and Procedural Modeling

Stelian Coros

Somewhat unrelated

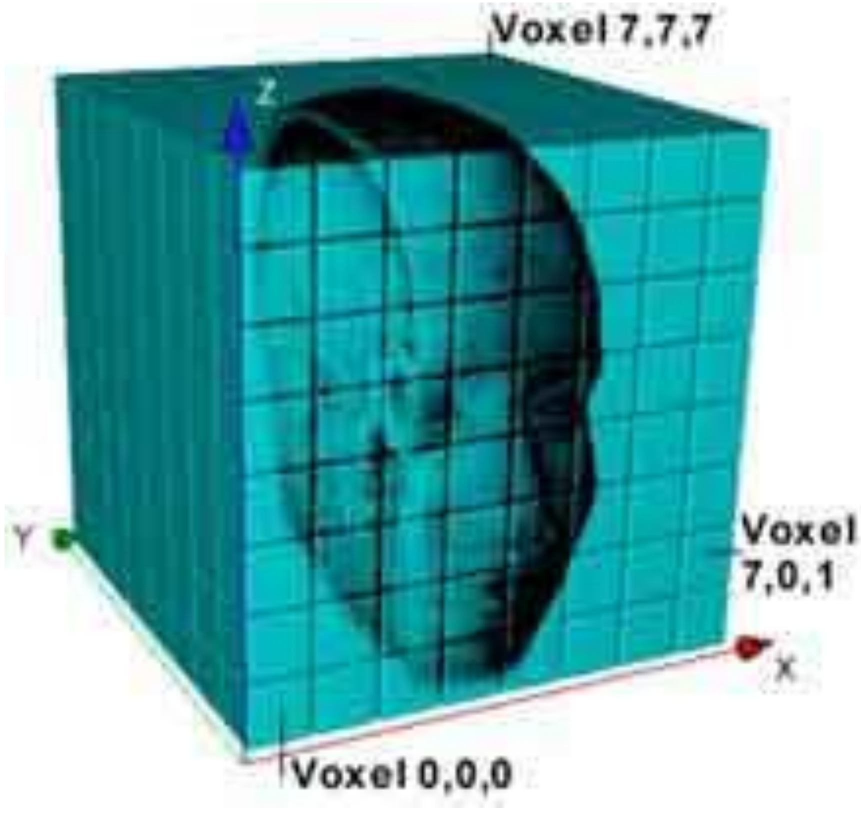
Schedule for presentations

February	3	5	10	12	17	19	24	26	
March	3	5	10	12	17	19	24	26	30
April	2	7	9	14	16	21	23	28	30

Send me:
ASAP: 3 choices for dates + approximate topic (scheduling)
1-2 weeks before your presentation: list of papers you plan to talk about
Day before each presentation: 3 questions for one of the papers that will be discussed

Previous Lecture: Solid Modeling

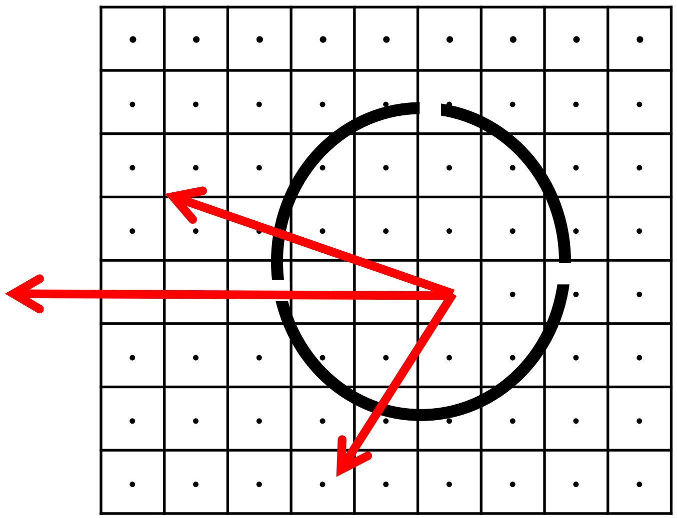
- Represent solid interiors of objects
  - Voxels
  - Octrees
  - Tetrahedra
  - Distance Fields



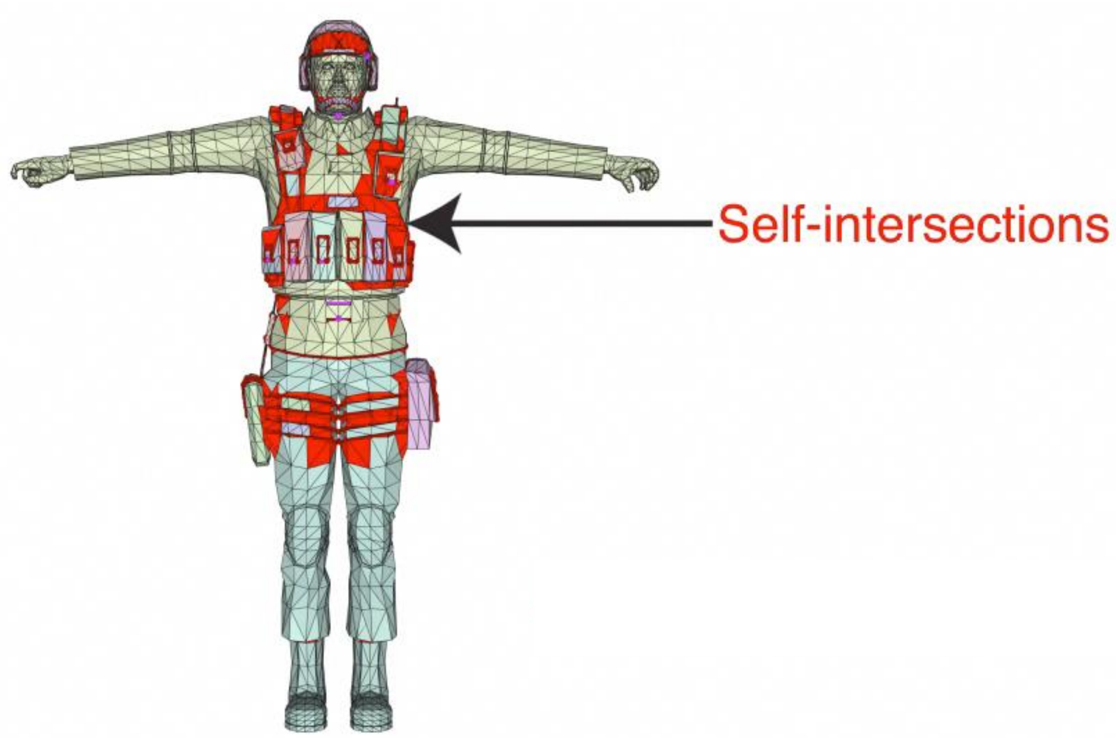
www.volumegraphics.com

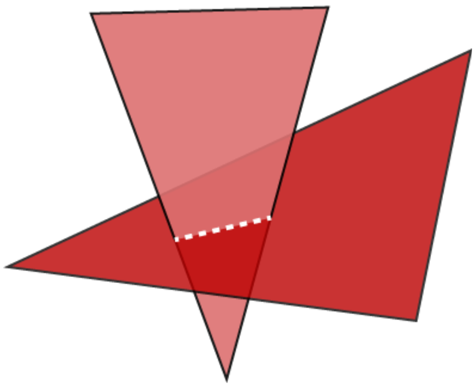
Previous Lecture: From Surfaces to Voxels

- Ray casting
  - Trace a ray from each voxel center
  - Count intersections
    - Odd: inside
    - Even: outside

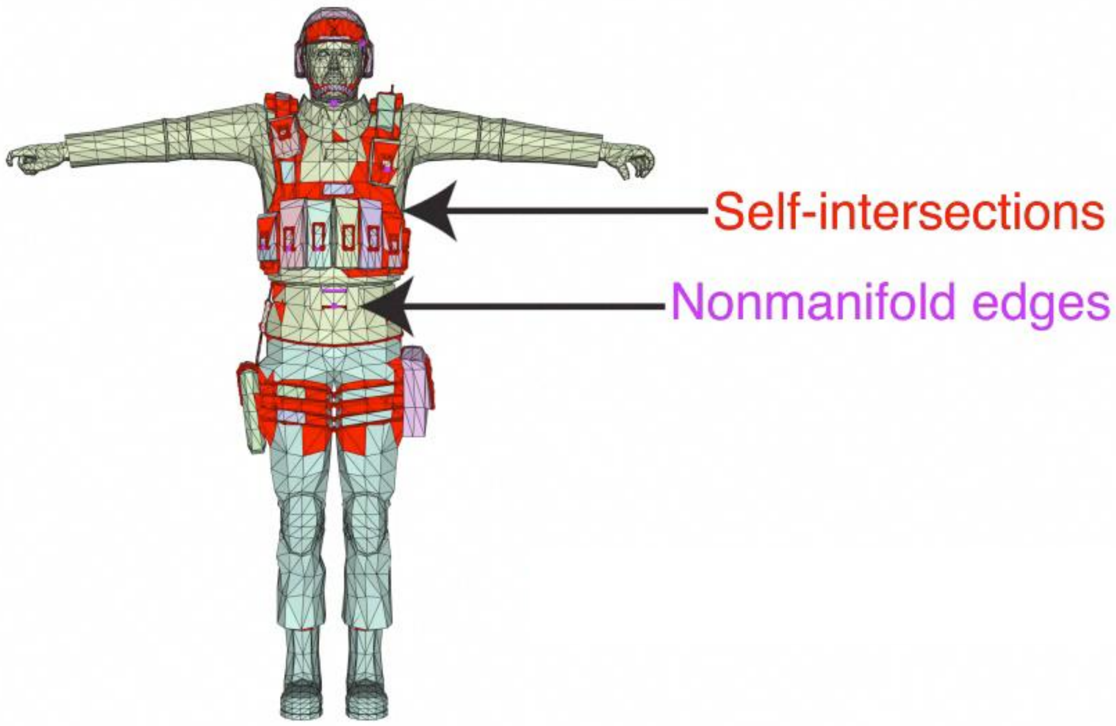


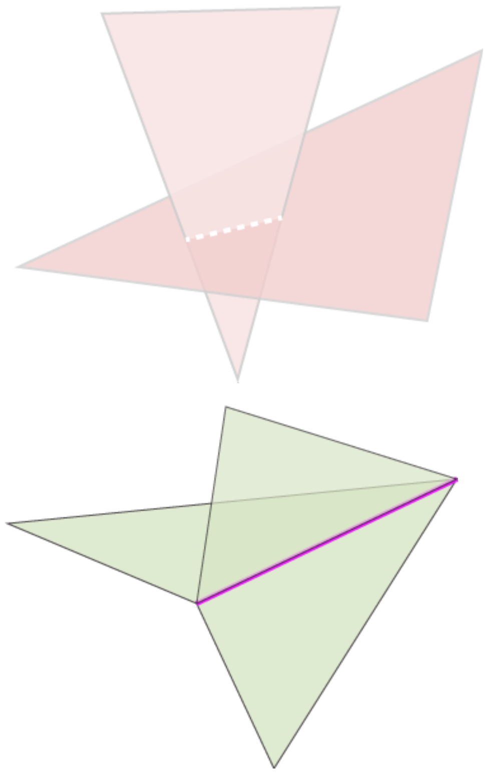
Real-life meshes



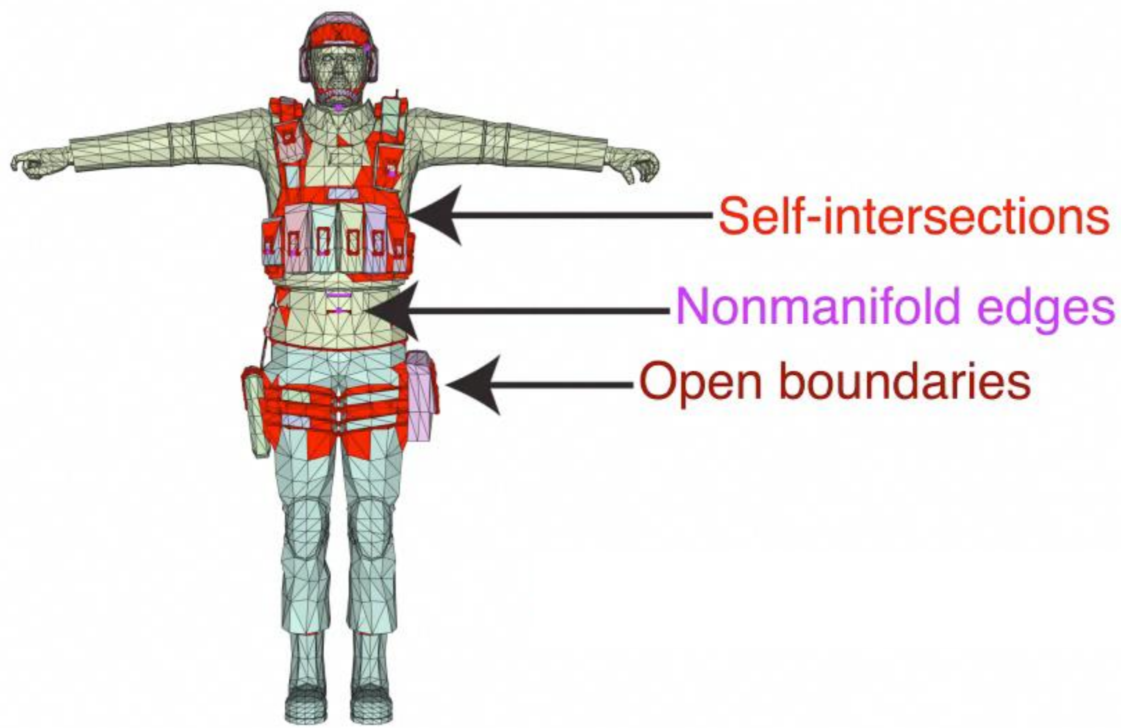


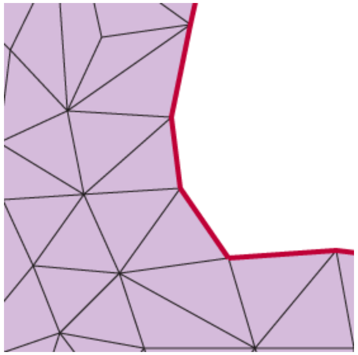
Real-life meshes



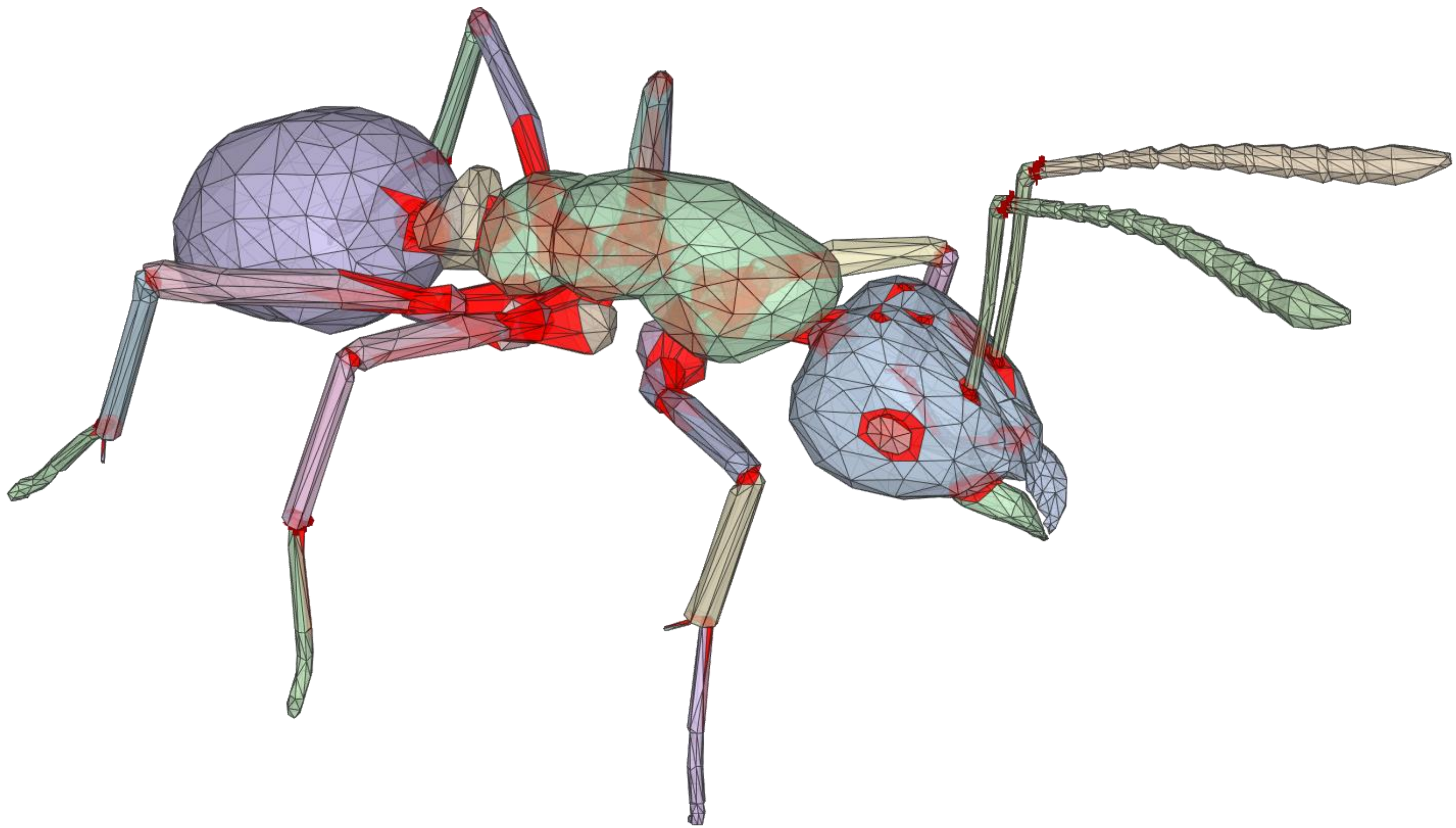


Real-life meshes





Real-life meshes: output of human creativity, for better or worse



Robust Inside-Outside Segmentation using Generalized Winding Numbers

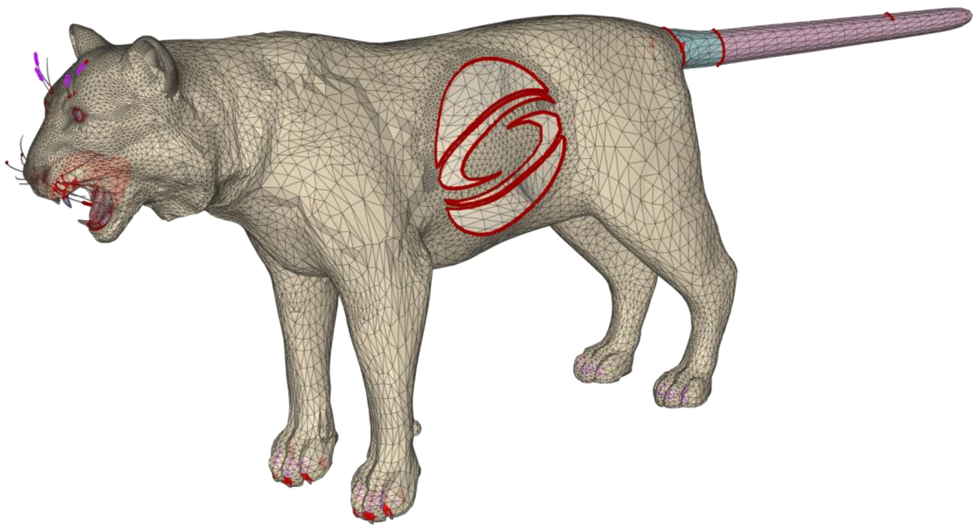
Alec Jacobson — ETH Zurich

Ladislav Kavan — University of Pennsylvania

Olga Sorkine-Hornung — ETH Zurich

Robust Inside-Outside Segmentation using Generalized Winding Numbers

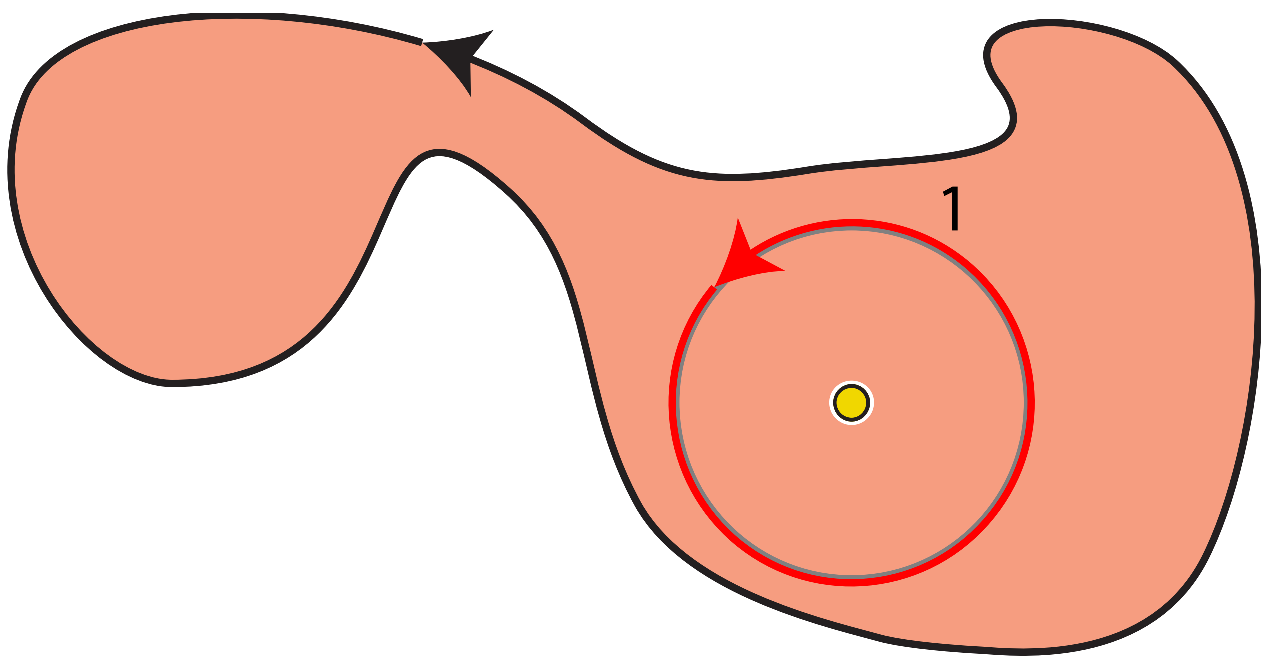
- Main challenge – determine which points are inside of a shape, which are outside



If shape is watertight, *winding number* is perfect measure of inside

- Winding number for a point in space:
  - how many times does the curve wind about the point
    Or, equivalently
  - Signed length of the curve projected on unit circle about the point

$$w(\mathbf{p}) = \frac{1}{2\pi} \oint_{\mathcal{C}} d\theta$$



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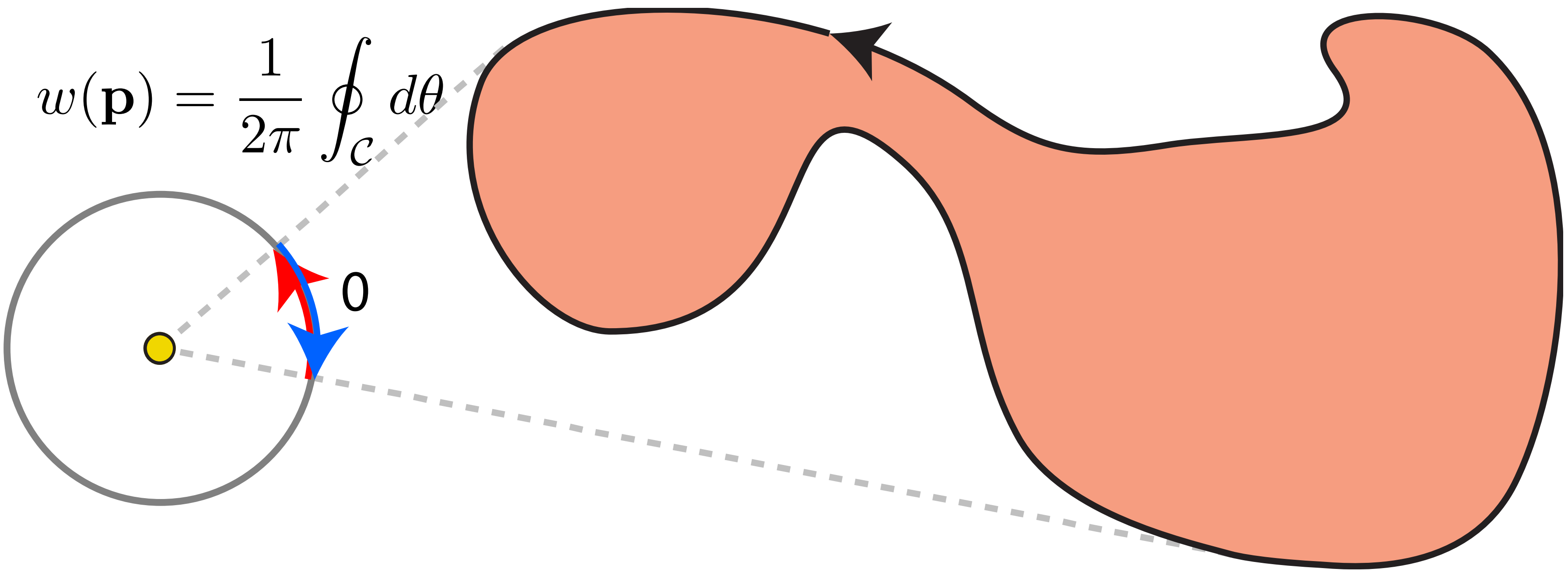
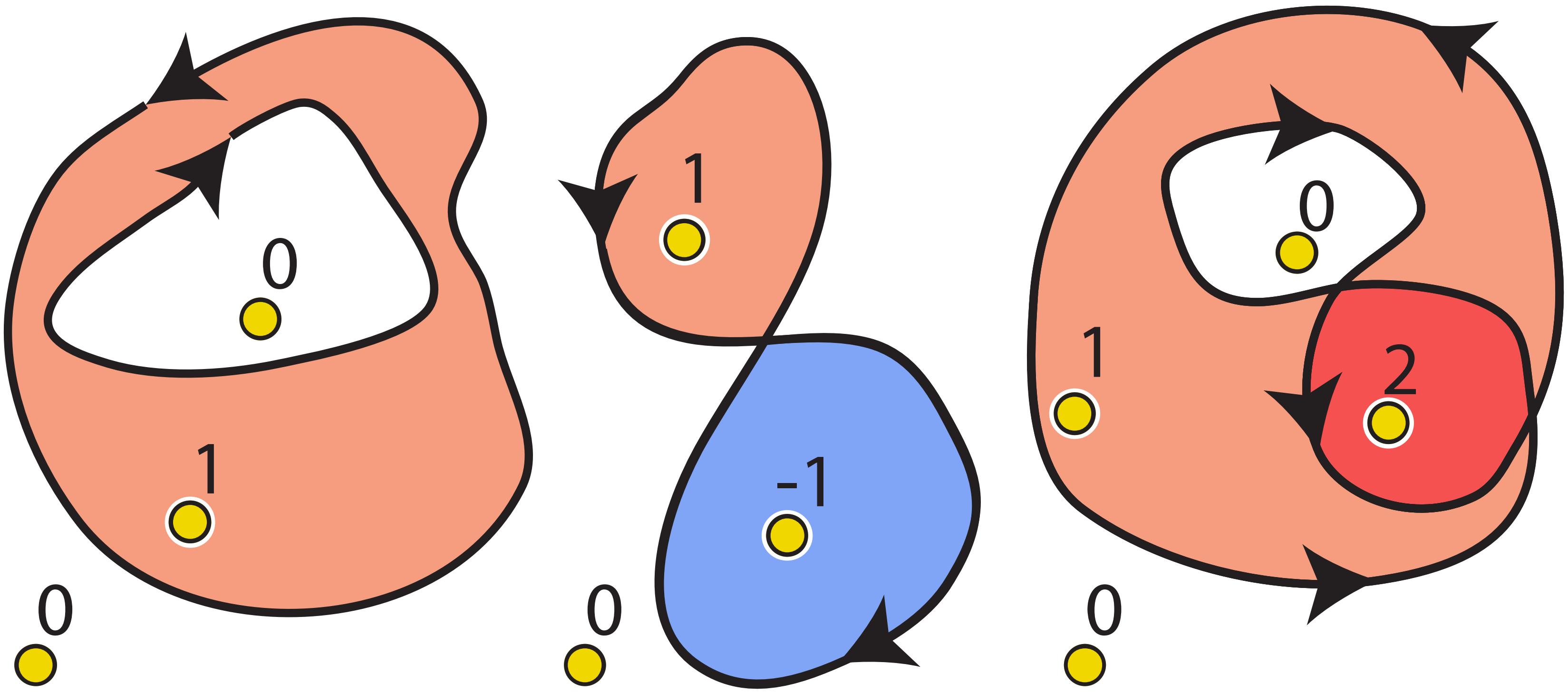


Robust for: arbitrary topologies, self-intersections, overlaps, and multiple connected components

- Use orientation of curve to treat *insideness* as integer quantity

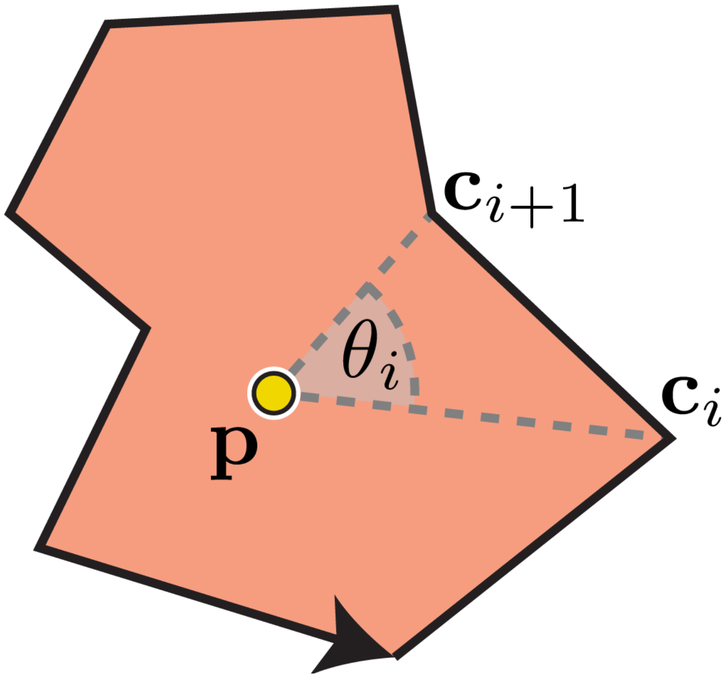


Winding number discretization (2D)

- Integral becomes sum of *signed* angle subtended by each edge

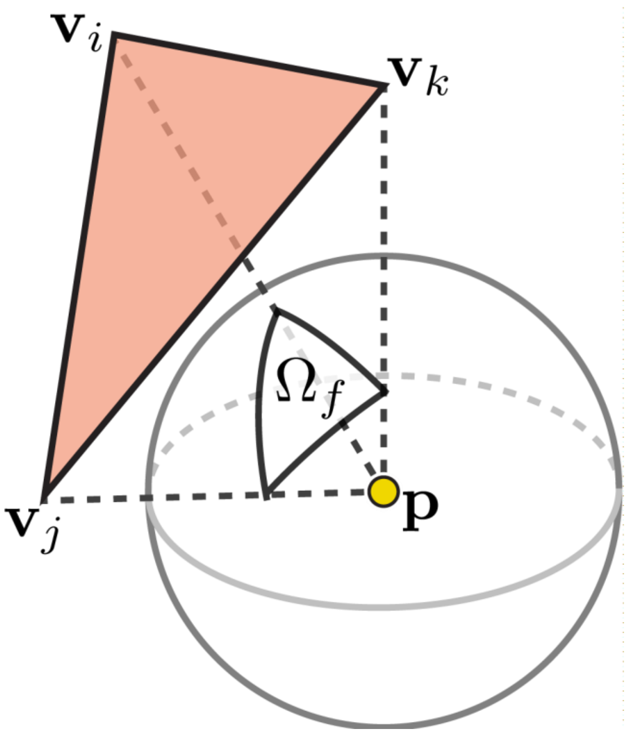
$$w(\mathbf{p}) = \frac{1}{2\pi} \oint_{\mathcal{C}} d\theta$$

$$w(\mathbf{p}) = \frac{1}{2\pi} \sum_{i=1}^{n} \theta_i$$



Winding number discretization (3D)

- *Solid* angle subtended by each triangle



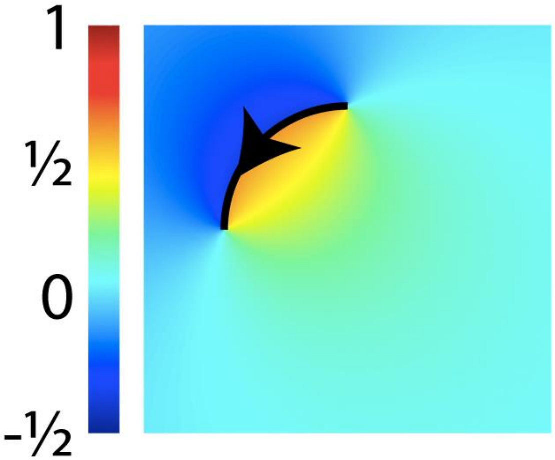
$$w(\mathbf{p}) = \frac{1}{4\pi} \iint_{S} \sin(\phi) d\theta d\phi$$

$$w(\mathbf{p}) = \frac{1}{4\pi} \sum_{f=1}^{m} \Omega_f$$

From nice meshes to real-world meshes

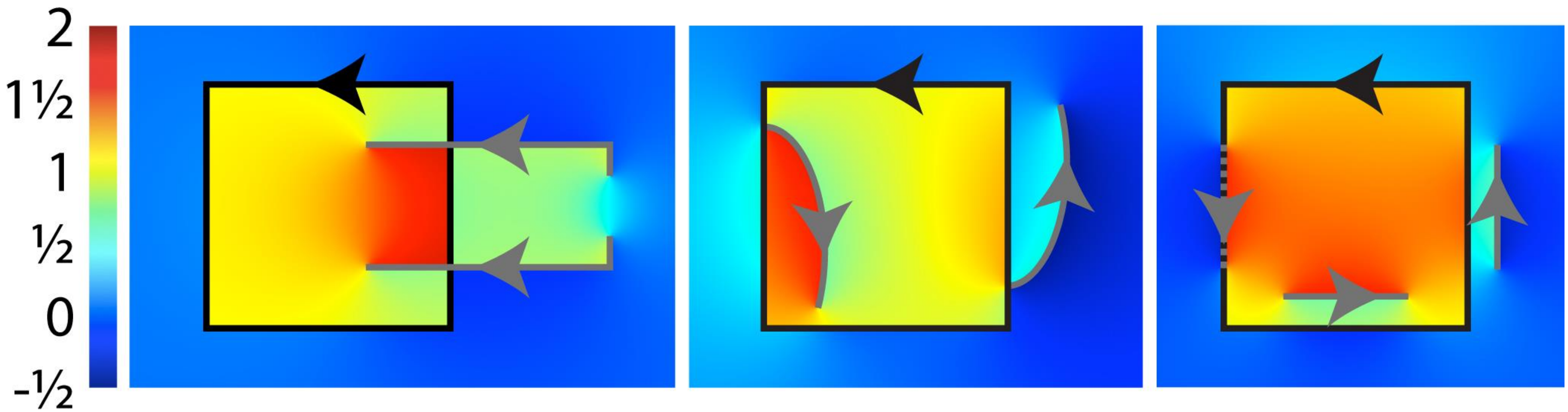
- Winding number no longer an integer value

$$w(\mathbf{p}) = \frac{1}{2\pi} \oint_{\mathcal{C}} d\theta$$



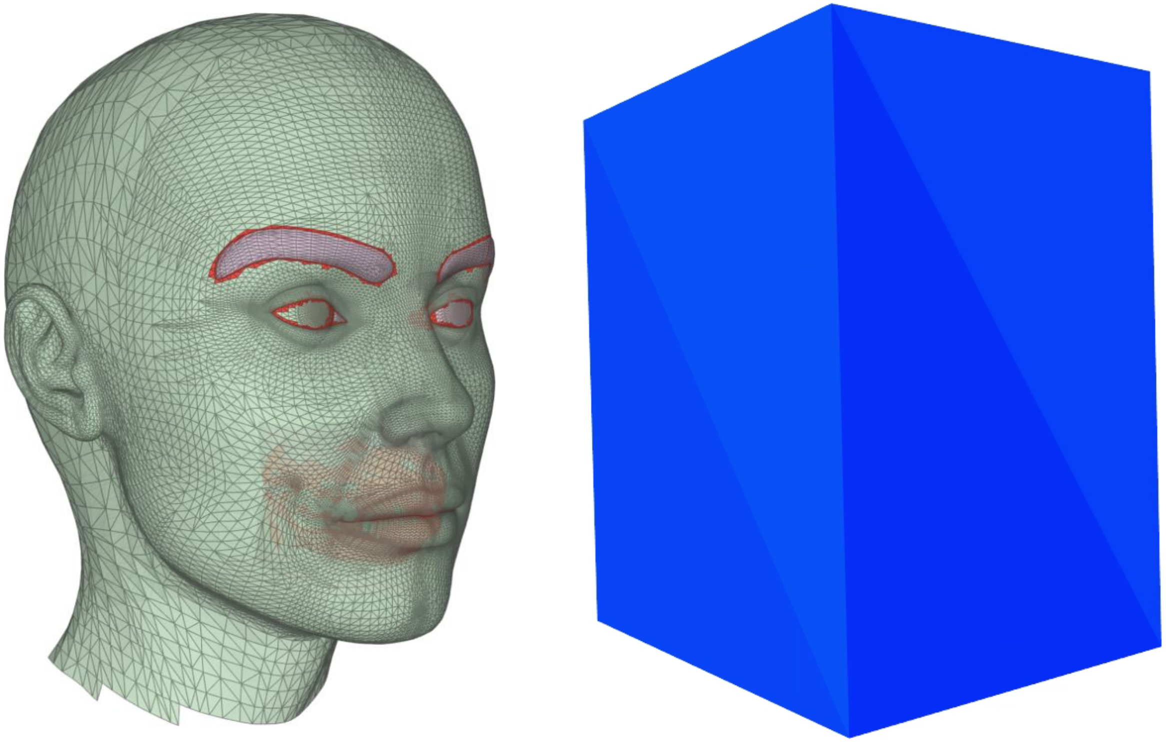
Gracefully tends toward perfect indicator as shape tends towards watertight

What if shape is self-intersecting? Non-manifold?

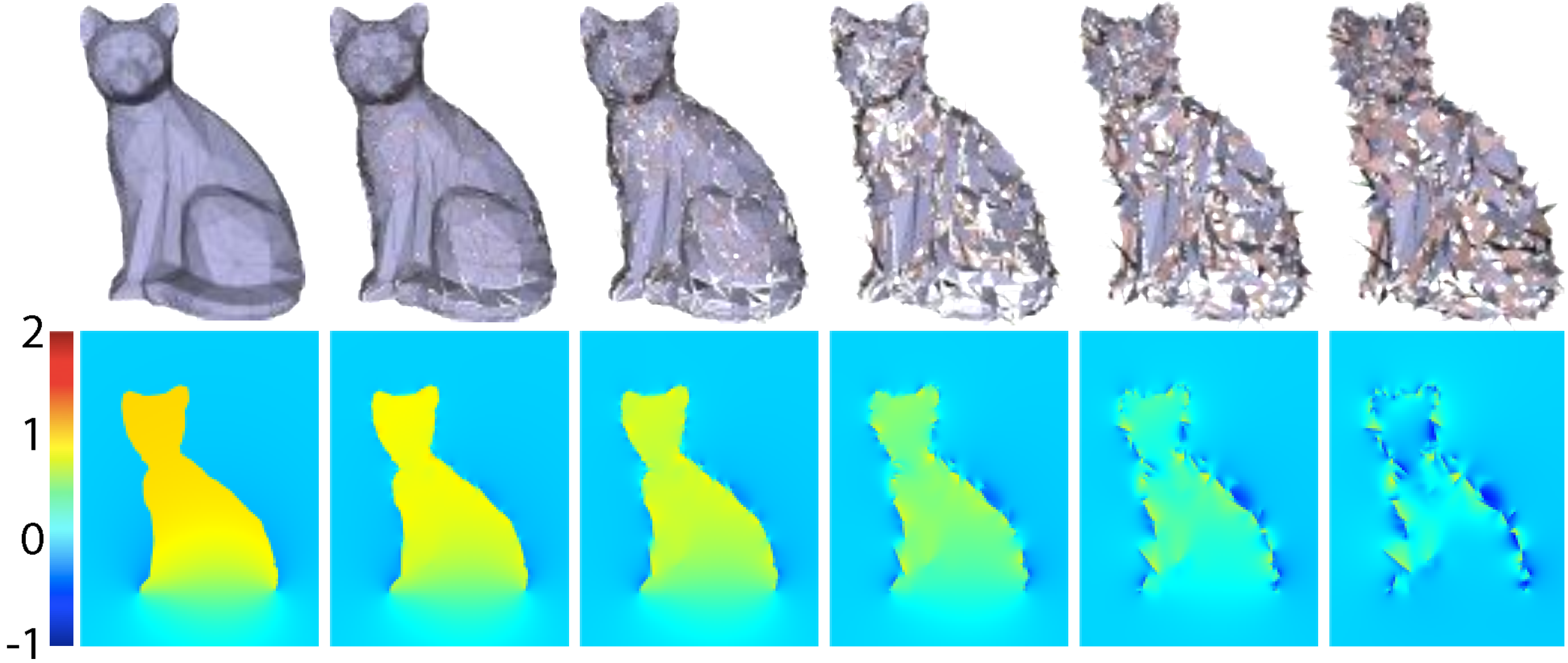


Normally smooth, jumps by ±1 across input facets

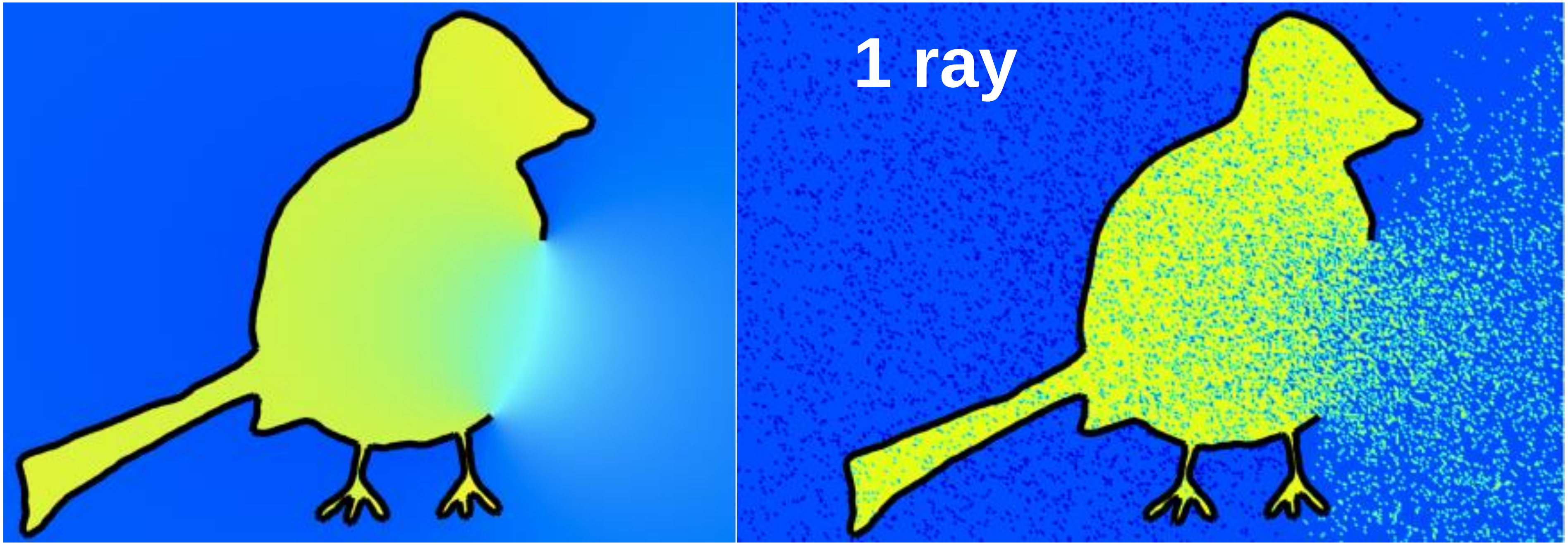
Sharp discontinuity across input eases precise segmentation



Winding number degrades gracefully



Winding number vs ray casting



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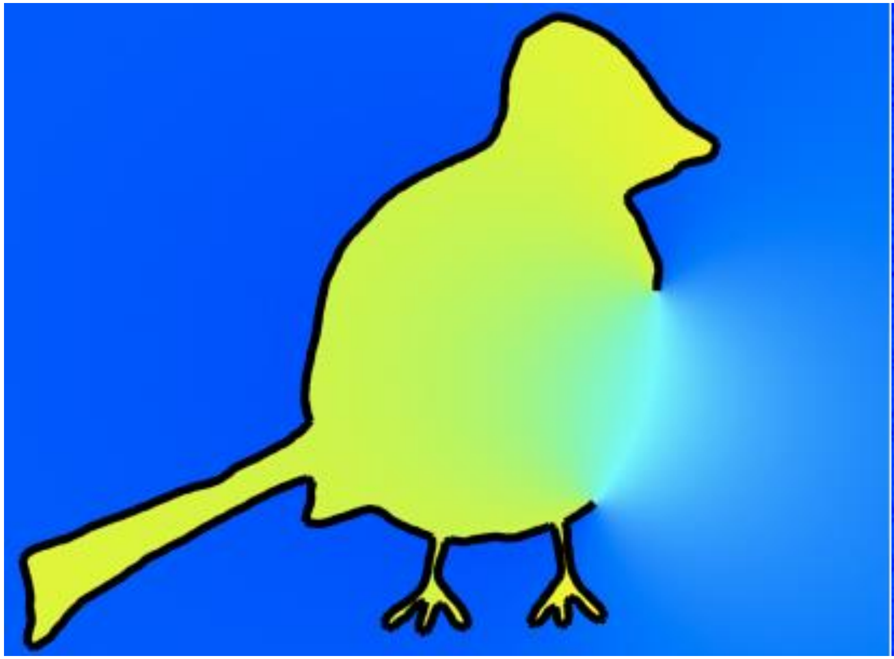

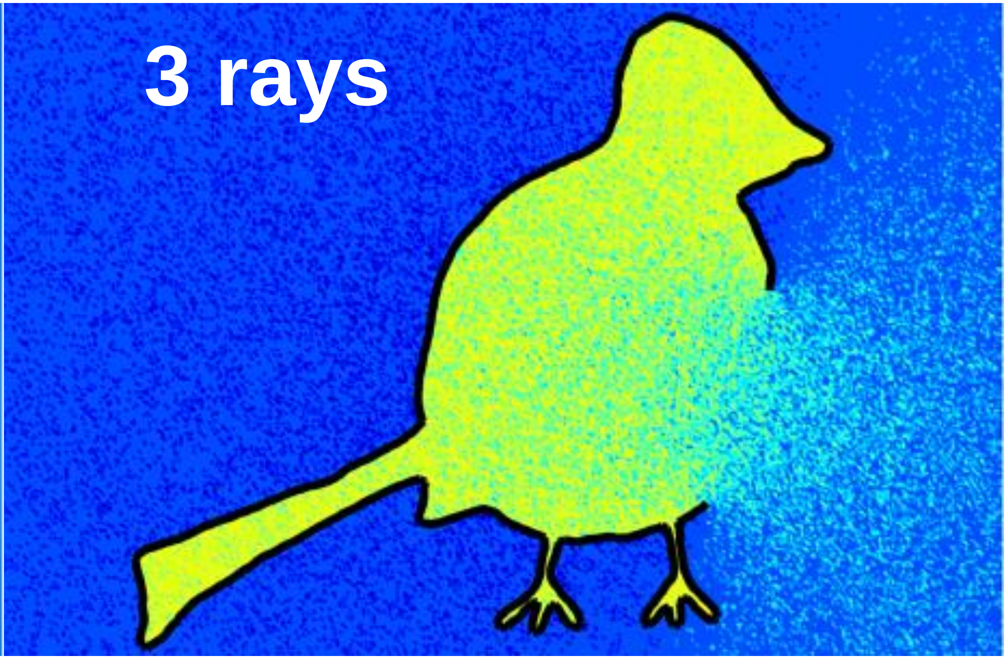

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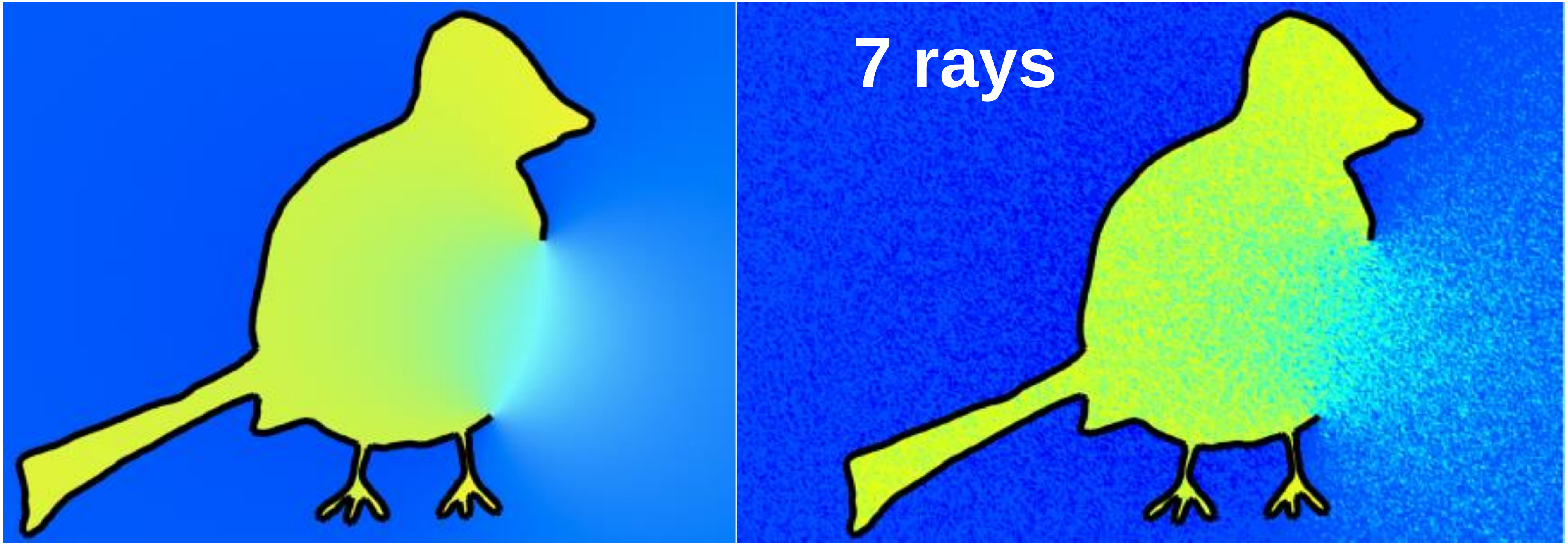
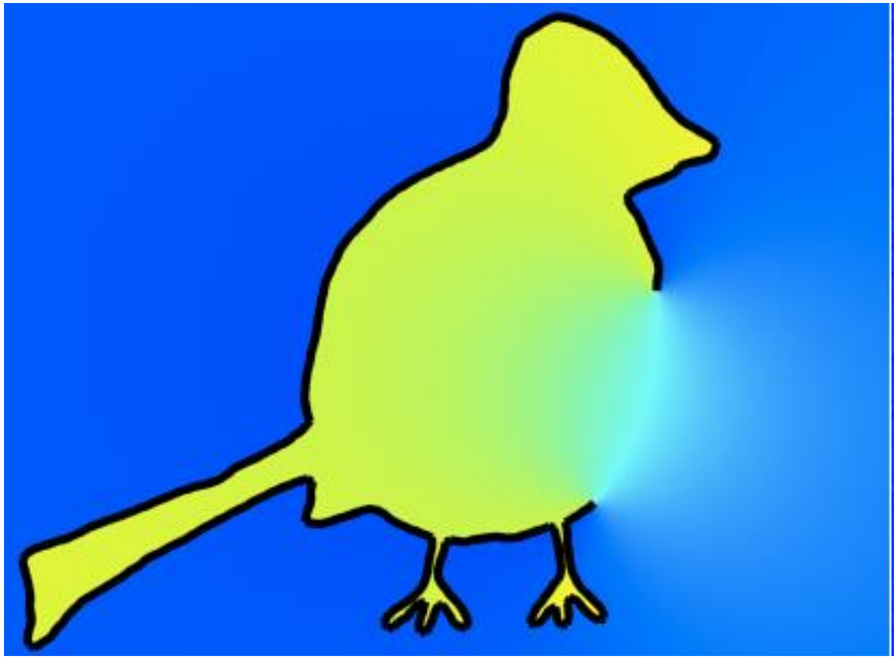
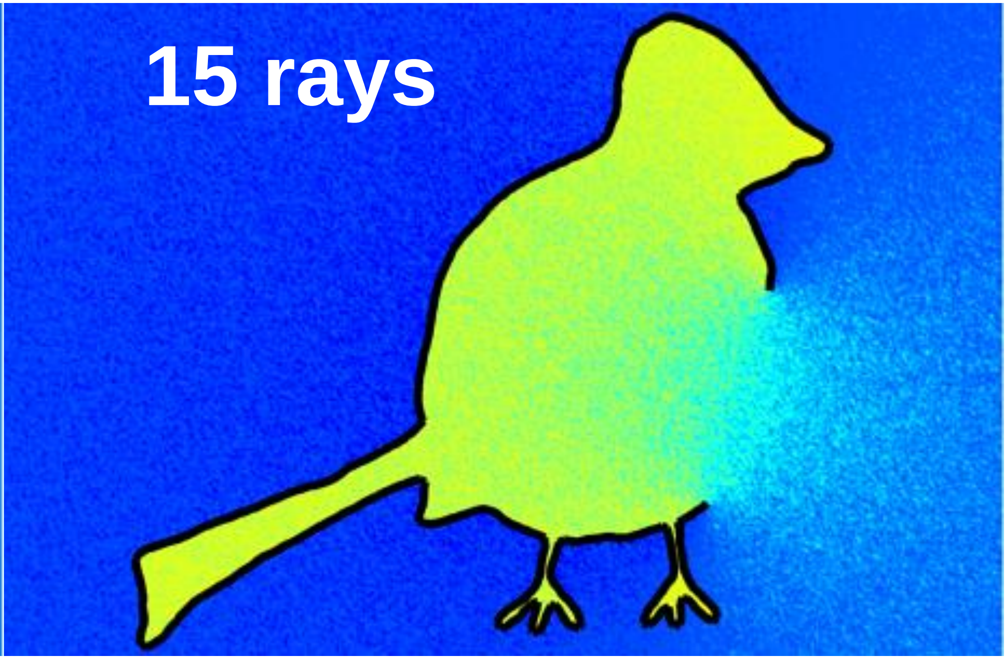
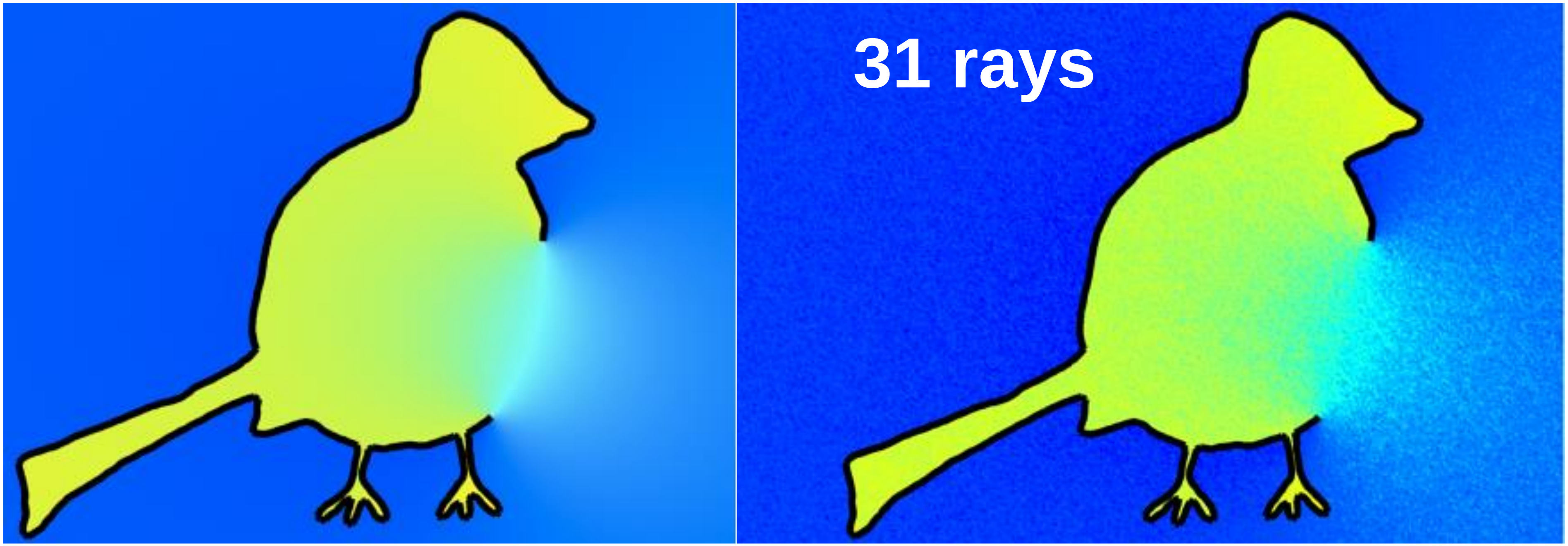


Winding number vs ray casting

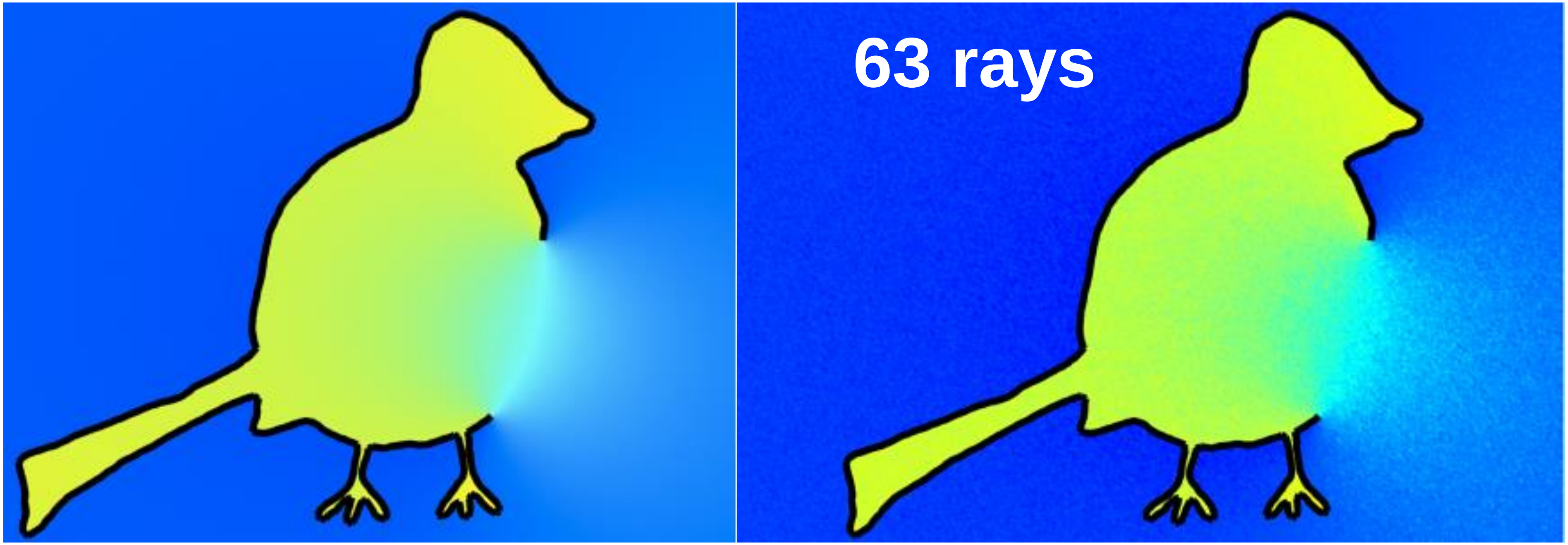




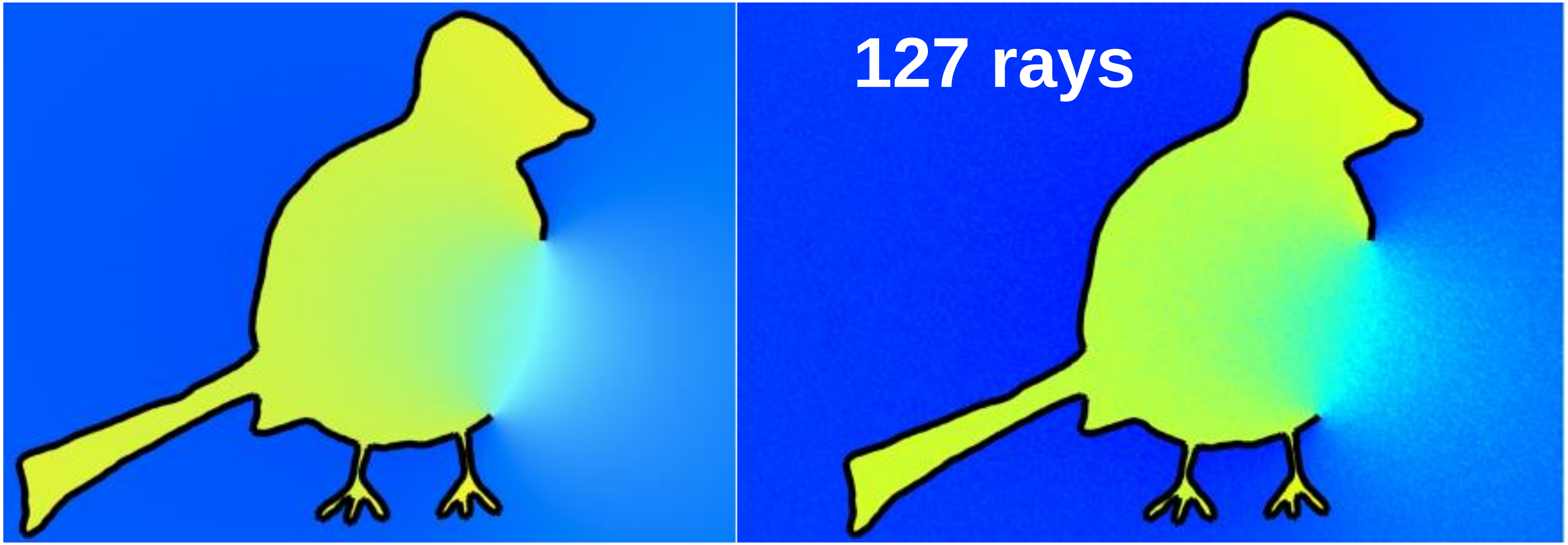
Winding number vs ray casting



Winding number vs ray casting



Winding number vs ray casting



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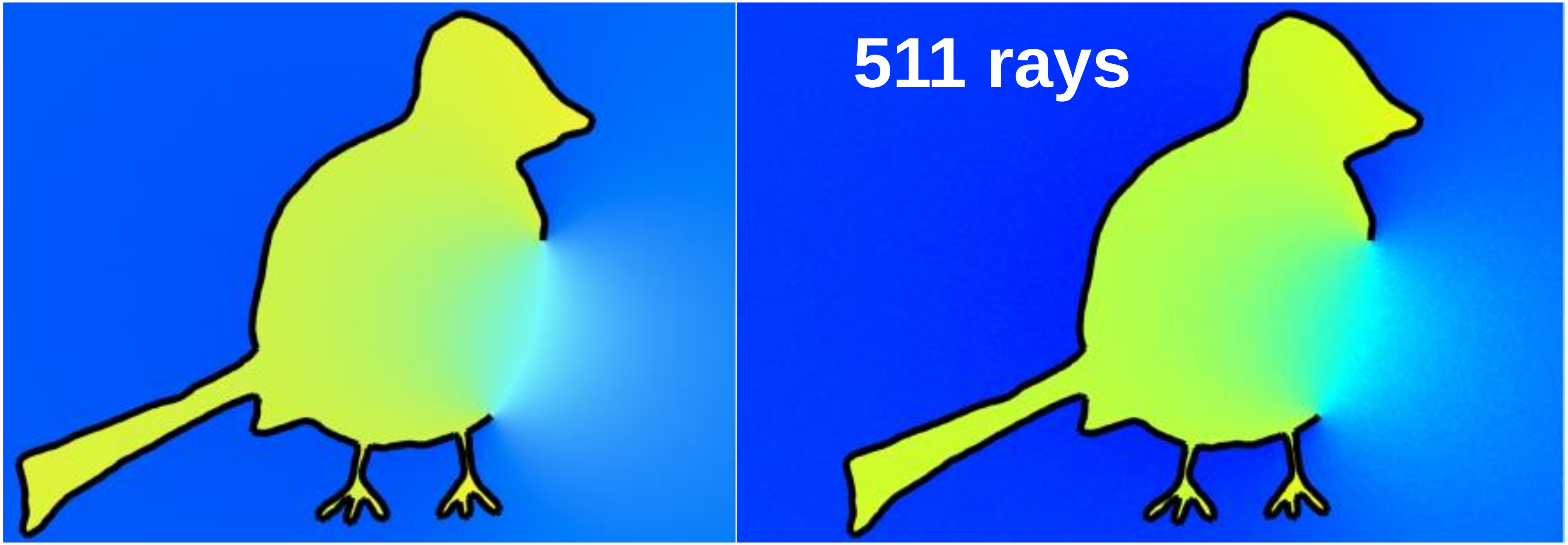
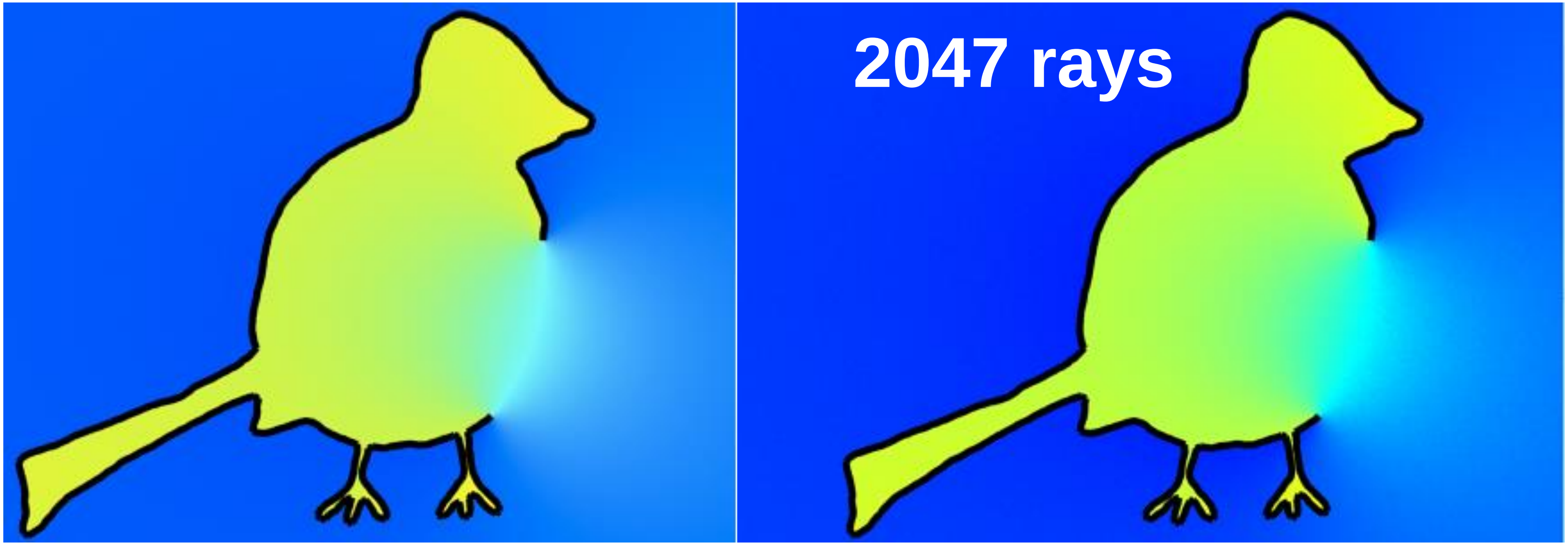


Winding number vs ray casting



Robust Inside-Outside Segmentation using Generalized Winding Numbers

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Geometric Representations

- Languages for describing shape
  - Boundary representations
    - Polygonal meshes
    - Subdivision surfaces
    - Implicit surfaces
  - Volumetric models

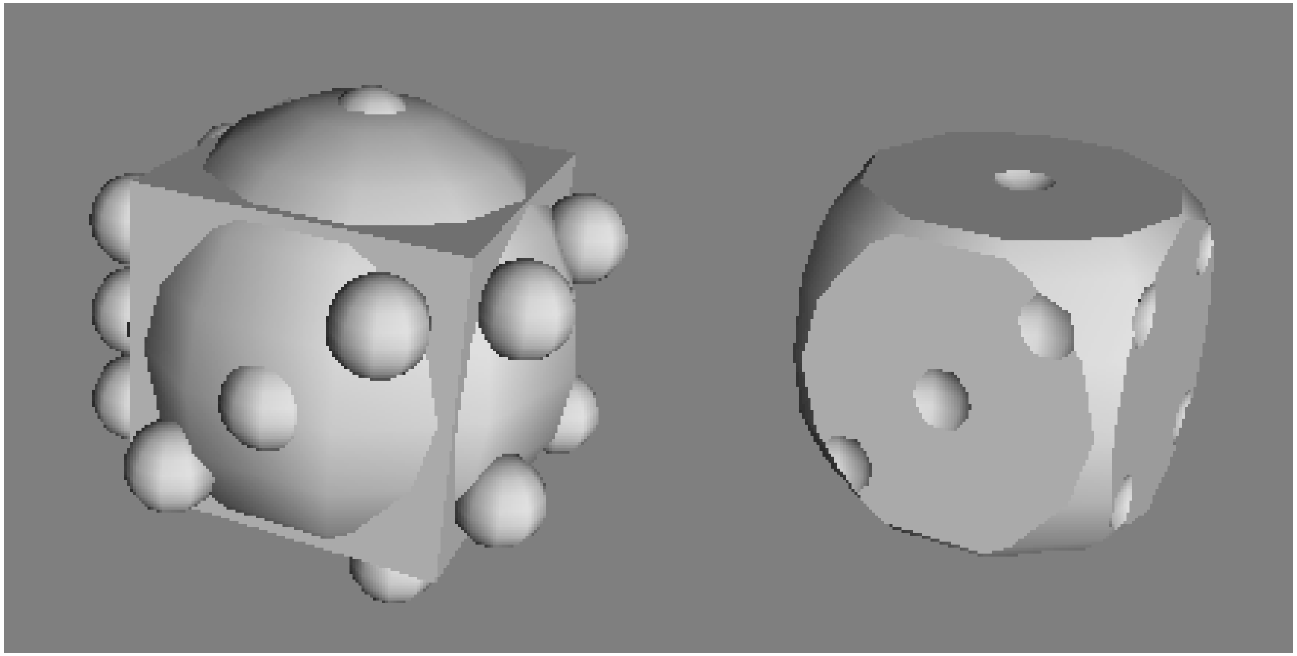
  - Parametric models
  - Procedural/generative models

Lower Level (Boundary representations, Volumetric models)

Higher Level (Parametric models, Procedural/generative models)

Constructive Solid Geometry (CSG)

- A way of building complex objects from simple primitives using Boolean operations



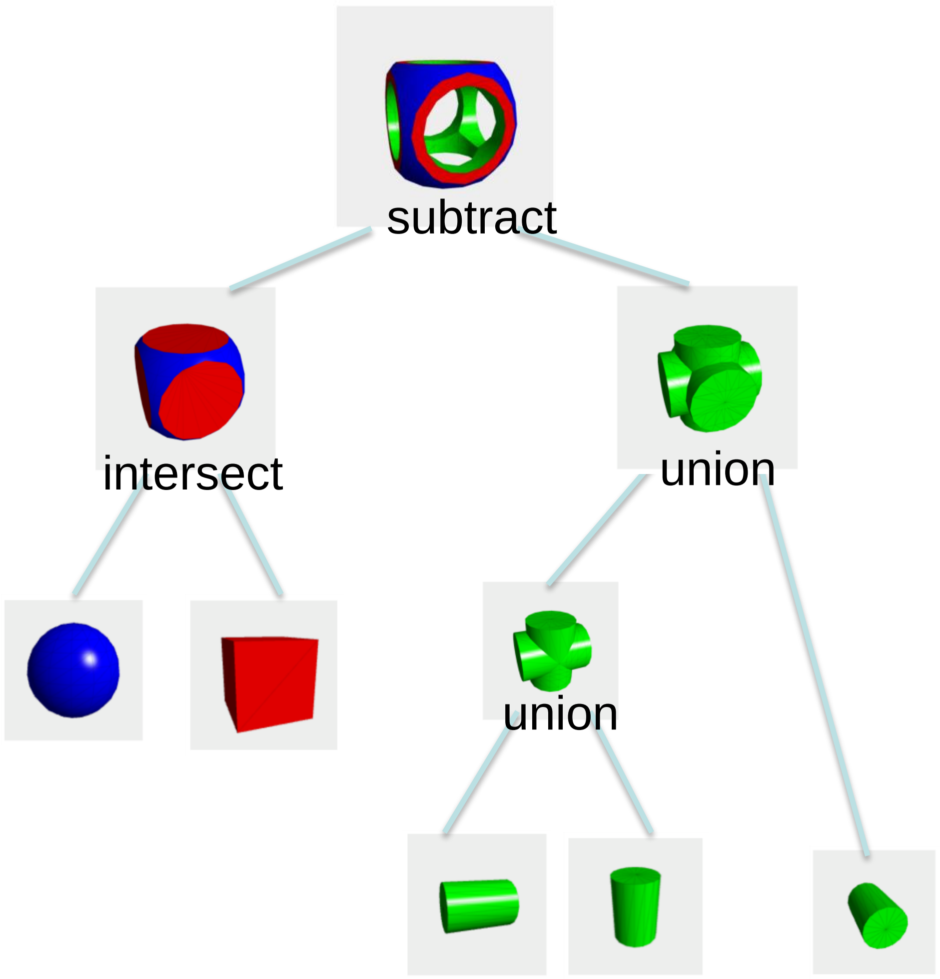
Constructive Solid Geometry (CSG)

- Represent solid object as hierarchy of Boolean operations
- The Boolean operations are not evaluated

CSG Data Structure

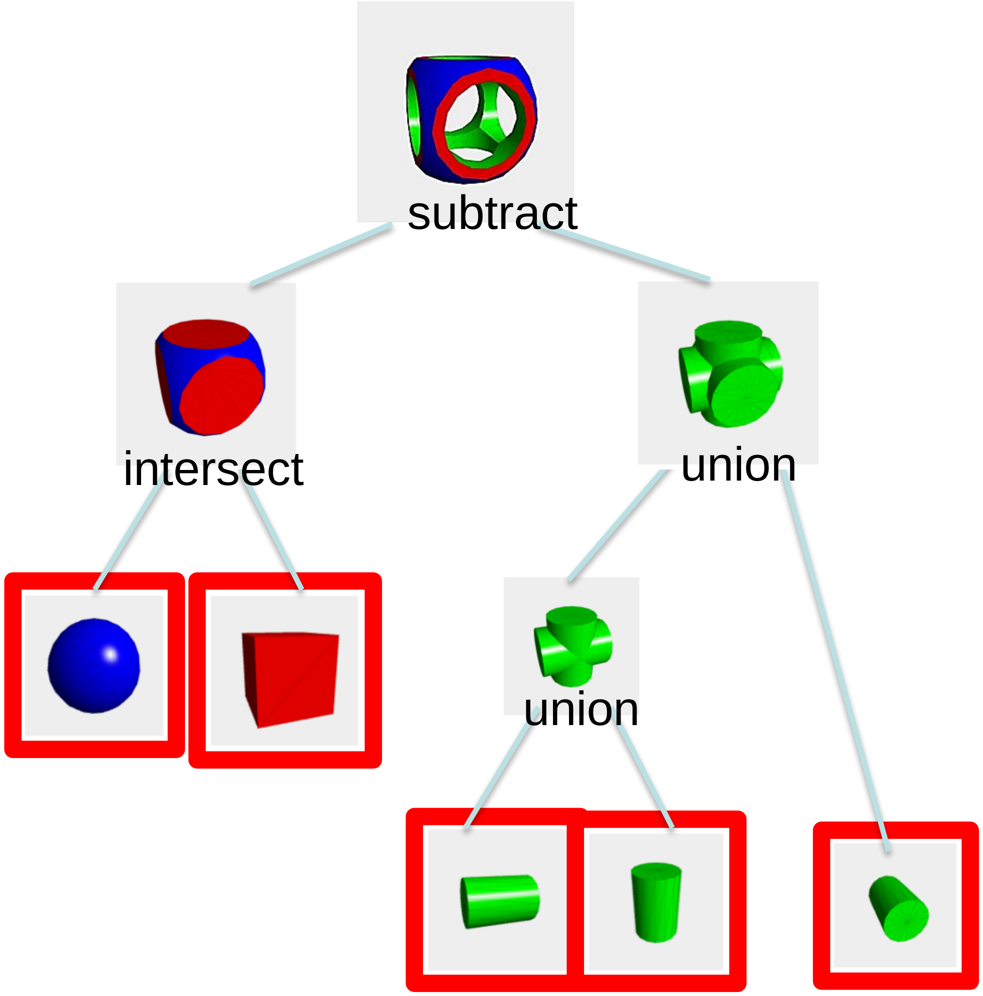
- Stored in a Binary Tree

  data structure



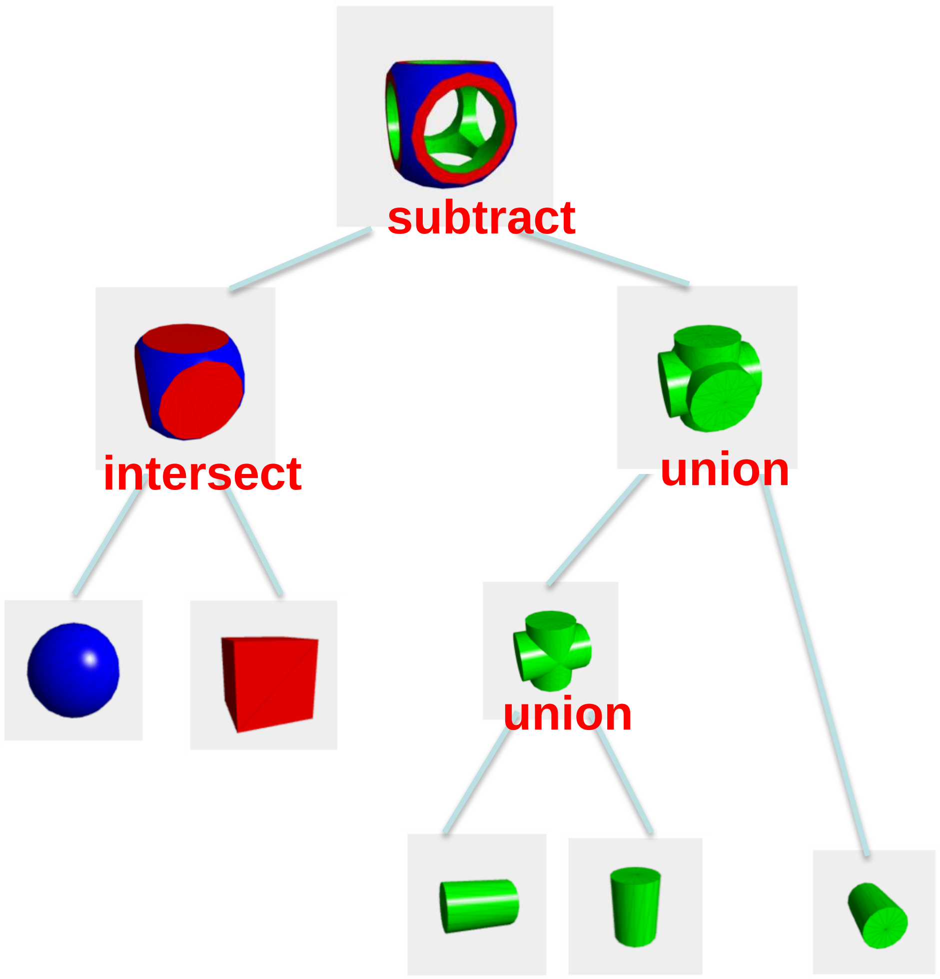
Leaves: CSG Primitives

- Simple shapes
  - Cuboids
  - Cylinders
  - Prisms
  - Pyramids
  - Spheres
  - Cones
- Extrusions
- Surfaces of revolution
- Swept surfaces

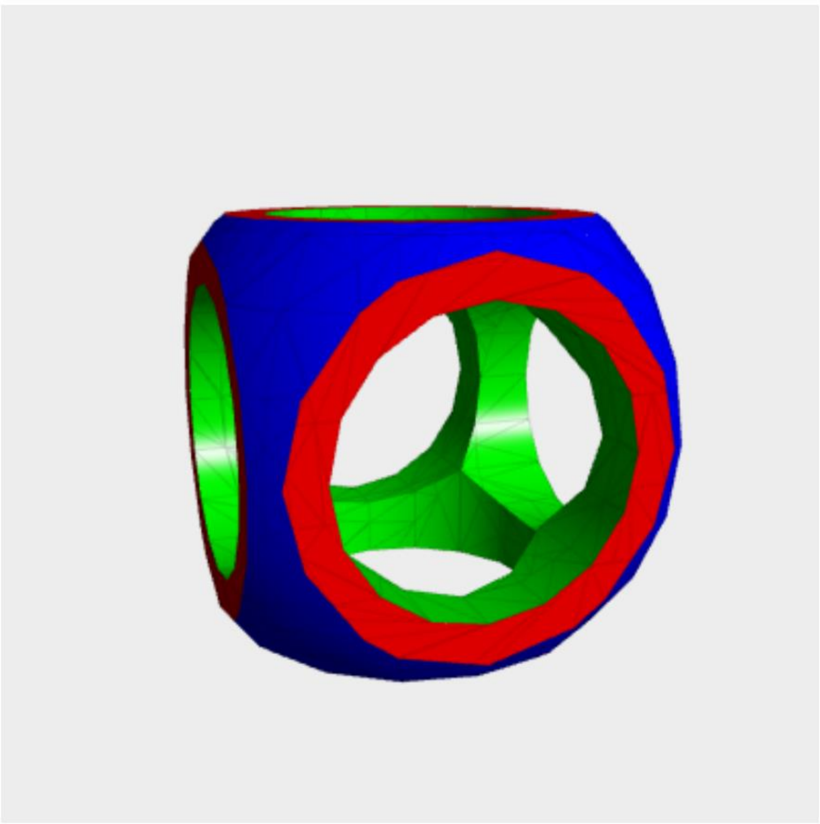


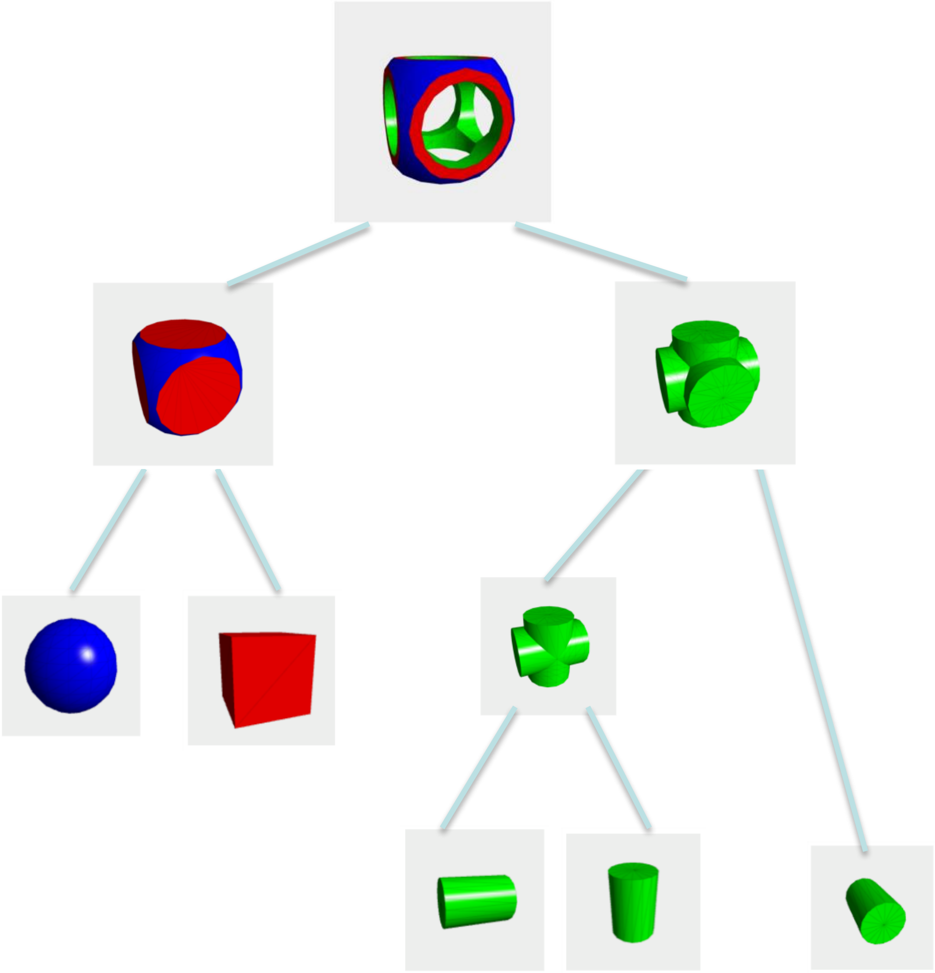
Internal Nodes

- Boolean Operations
  - Union
  - Intersection
  - Difference
- Rigid Transformations
  - Scale
  - Translation
  - Rotation
  - Shear



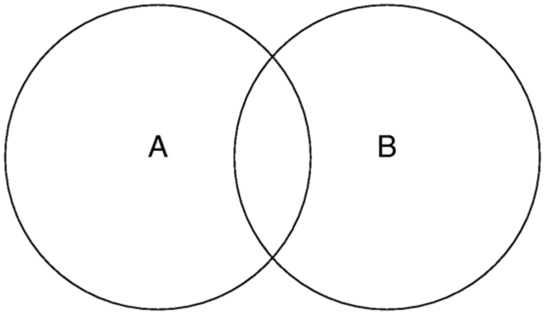
Root: The Final Object



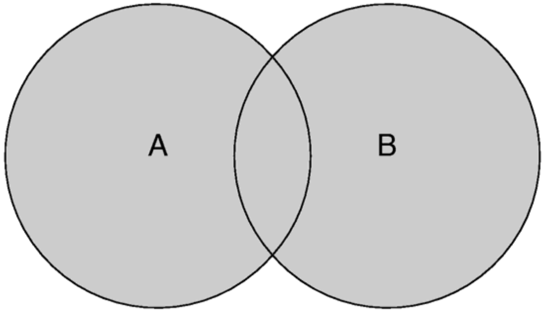


Booleans for Solids

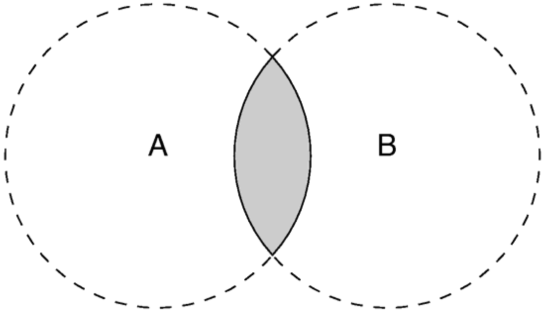
Given overlapping shapes A and B:



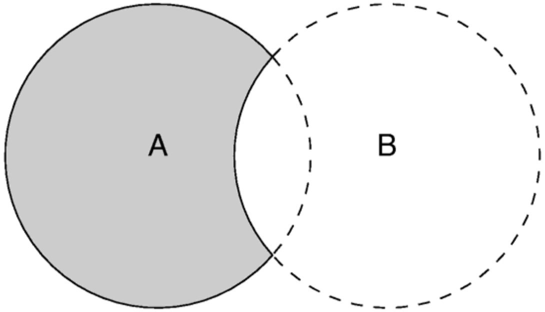
Union



Intersection

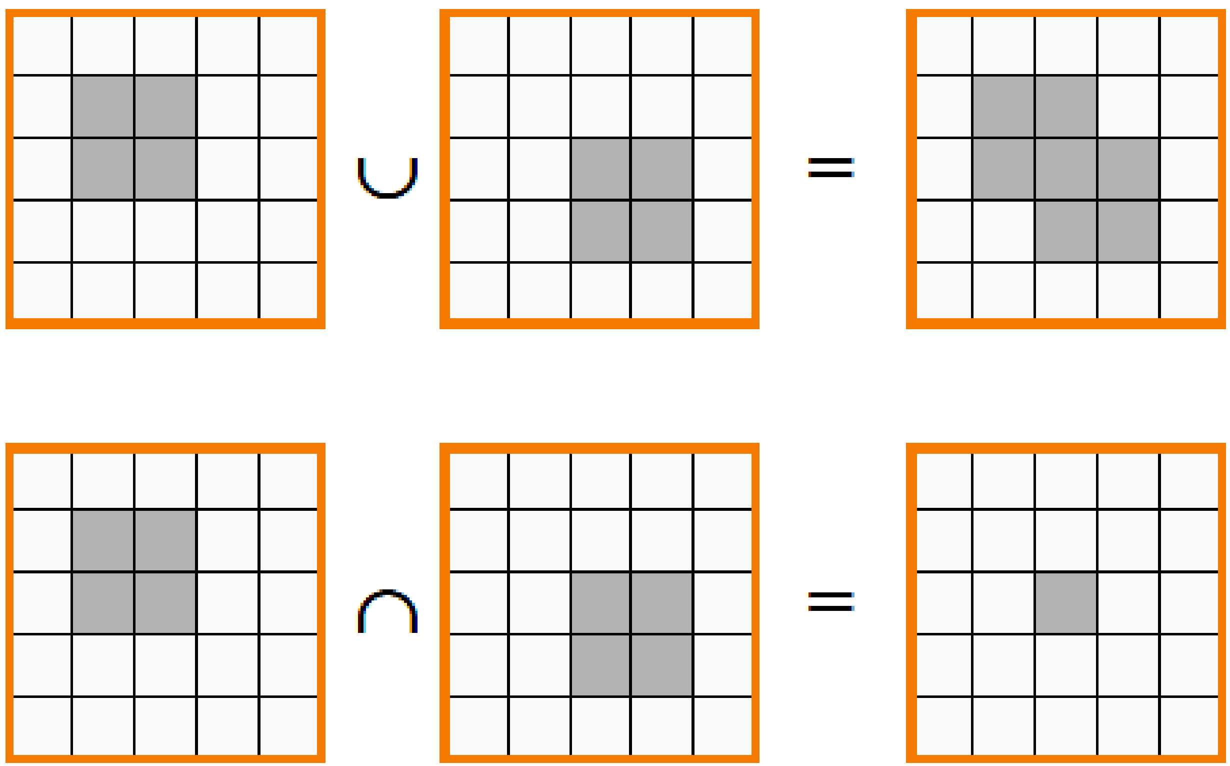


Subtraction



How Can We Implement Boolean Operations?

- Use voxels/octrees/ADFs
  - We can convert from b-reps to voxels/DF and back
  - Process objects voxel by voxel
  - Issues?

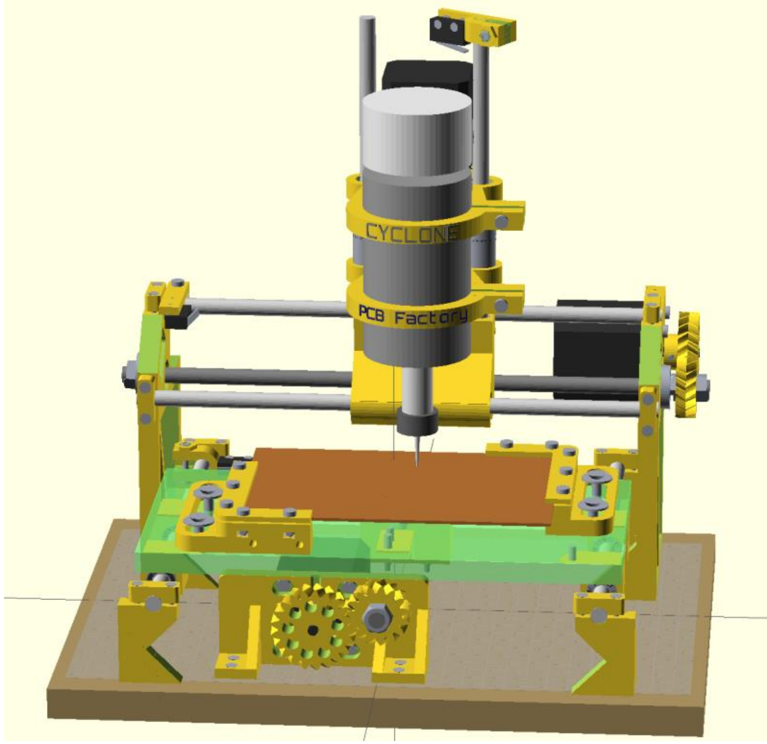


How Can We Implement Boolean Operations?

- Directly: the hard way ...
  - You will not be asked to do this
- Commercial libraries/CAD tools
  - e.g., Parasolid, SolidWorks
- Open source libraries
  - e.g., CGAL, OpenSCAD

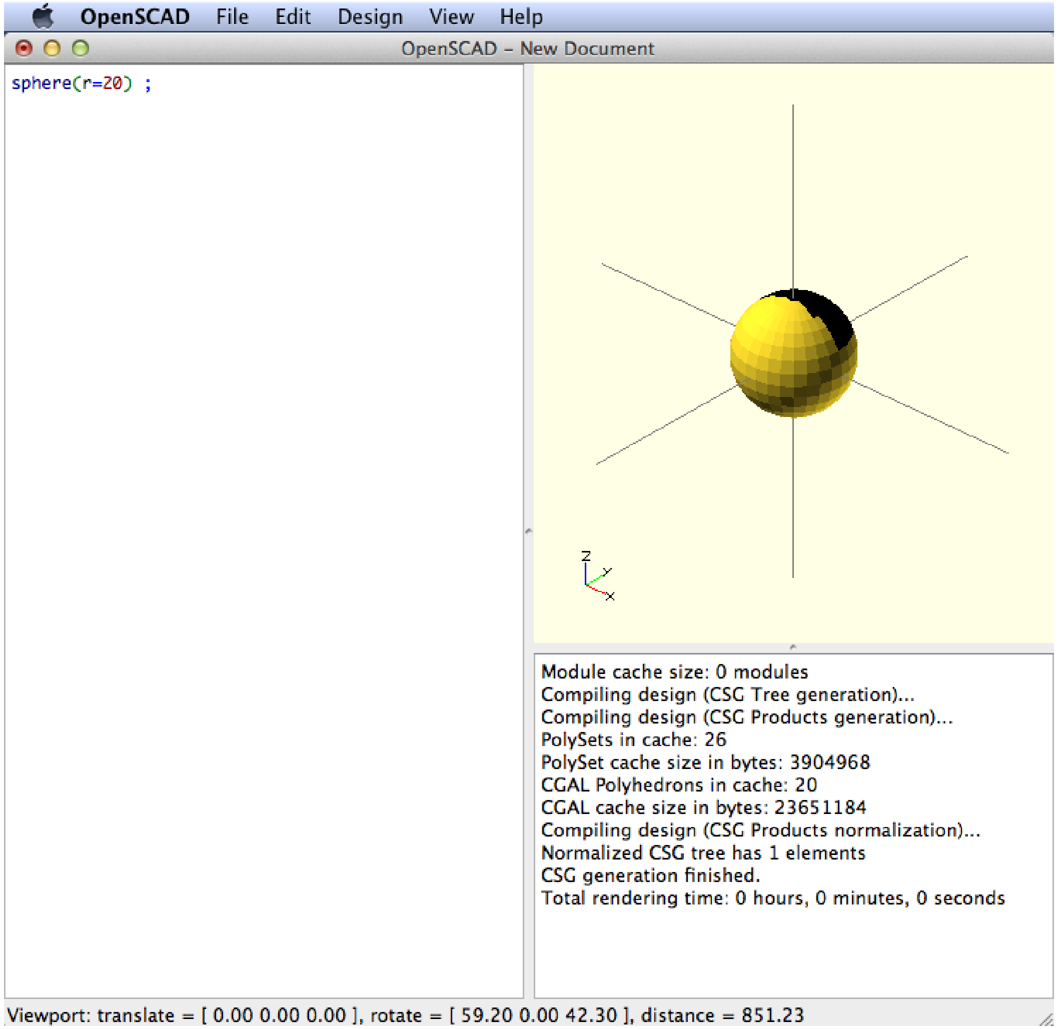
OpenSCAD

- Software for creating solid 3D CAD models
- Not an interactive modeler
  - Very basic UI
- A 3D-compiler
  - Geometry written as a script
  - Executed using CGAL/OpenCSG
  - Rendered with OpenGL
- Available for Linux/UNIX, Windows, Mac OS X
  - http://www.openscad.org



OpenSCAD

- Interface
  - 3 panels
    - Script
    - View
    - Info
- Compile (F5)
  - Design->Compile
- Show Axes (Ctrl+2)



OpenSCAD CheatSheet

OpenSCAD CheatSheet

Syntax

var = value;

module name(…) { … }
name();

function name(…) = …
name();

include <….scad>

use <….scad>

2D

circle(radius)

square(size,center)

square([width,height],center)

polygon([points])

polygon([points],[paths])

3D

sphere(radius)

cube(size)

cube([width,height,depth])

cylinder(h,r,center)

cylinder(h,r1,r2,center)

polyhedron(points, triangles, convexity)

Transformations

translate([x,y,z])

rotate([x,y,z])

scale([x,y,z])

resize([x,y,z],auto)

mirror([x,y,z])

multmatrix(m)

color("colorname")

color([r, g, b, a])

hull()

minkowski()

Boolean operations

union()

difference()

intersection()

Modifier Characters

*	disable
!	show only
#	highlight
%	transparent

Mathematical

abs

sign

acos

asin

atan

atan2

sin

cos

floor

round

ceil

ln

len

log

lookup

min

max

pow

sqrt

exp

rands

Other

echo(…)

str(…)

for (i = [start:end]) { … }

for (i = [start:step:end]) { … }

for (i = […,…,…]) { … }

intersection_for(i = [start:end]) { … }

intersection_for(i = [start:step:end]) { … }

intersection_for(i = […,…,…]) { … }

if (…) { … }

assign (…) { … }

search(…)

import("….stl")

linear_extrude(height,center,convexity,twist,slices)

rotate_extrude(convexity)

surface(file = "….dat",center,convexity)

projection(cut)

render(convexity)

Special variables

$fa minimum angle

$fs minimum size

$fn number of fragments

$t animation step

Links

- Official website
- Manual
- MCAD library
- Other links

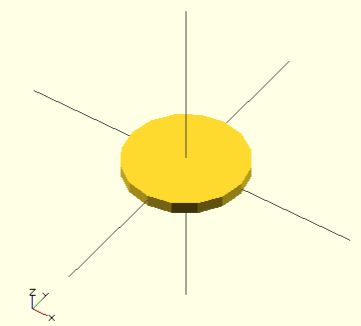
Examples

cylinder(10,5,5);

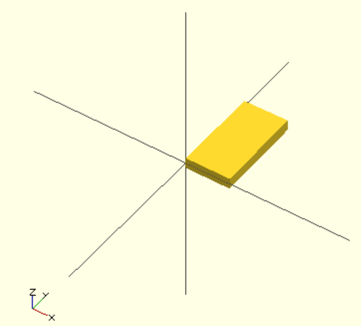
cylinder(h=10,r=5);

2D Primitives

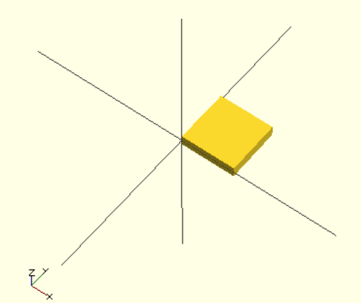
- Circle
  - `circle(5);`
  - `circle(r=5);`



- Square
  - `square(5);`
  - `square([4,8]);`



- Polygon
  - Need to specify points and paths, in this format: polygon([points],[paths]);
    - e.g., `polygon( [ [0,0],[5,0],[5,5],[0,5] ] , [ [0,1,2,3] ]);`
    - path is an optional parameter, assume in order if omitted

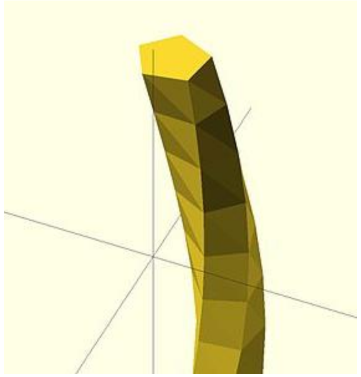


- Notes:
  - Remember the “;”
  - Thickness is 1mm
  - Use “[“ and “]” to pass multiple values

2D to 3D Extrusion

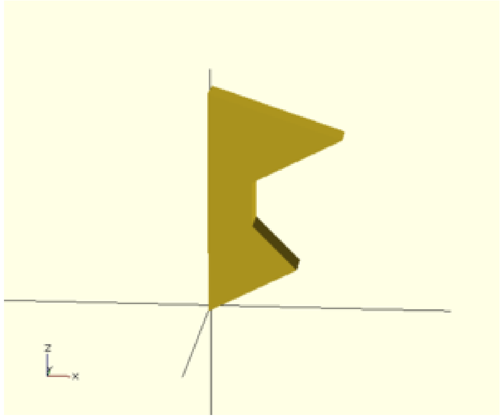
- Linear extrusion
  - Extrudes a 2D shape along the Z axis

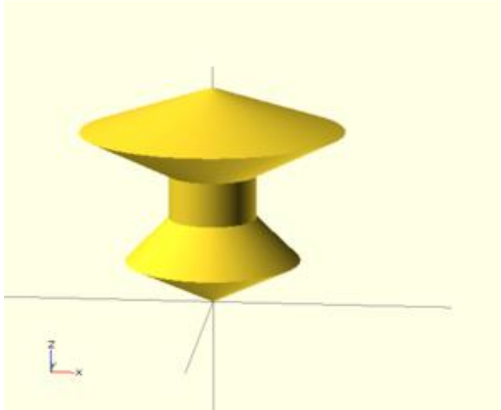
```
linear_extrude(height = 10, center = true, convexity = 10, twist = -100) translate([2, 0, 0]) circle(r = 1);
```



- Rotational extrusion
  - Revolves a 2D shape around the Z axis

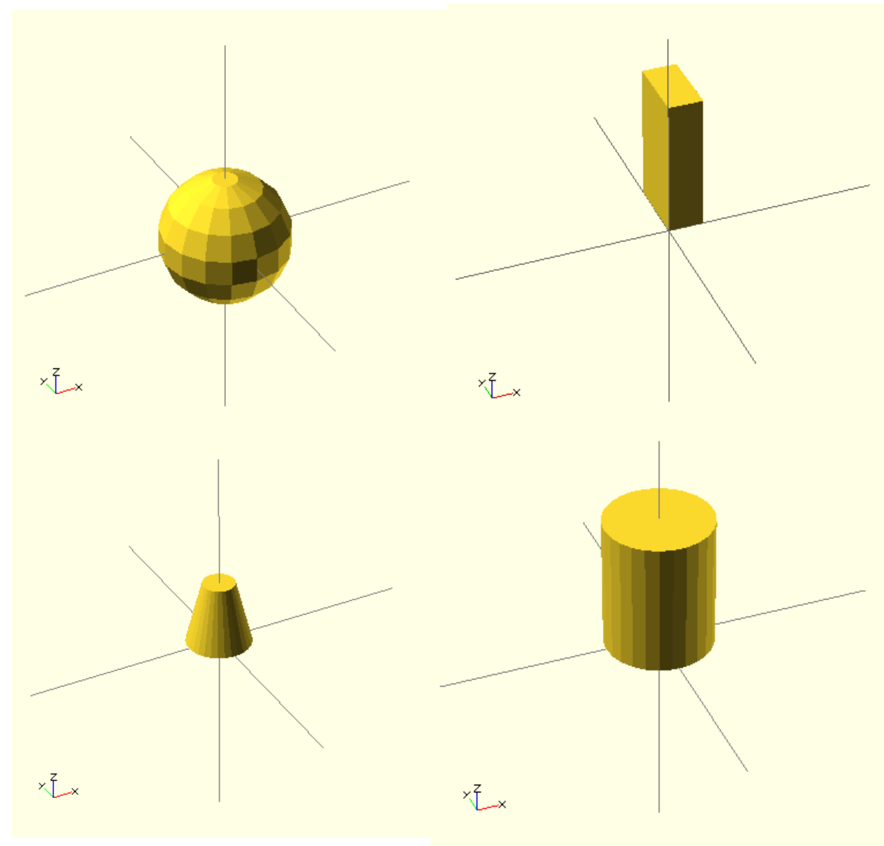
```
rotate_extrude($fn=200)
polygon( points=[[0,0],[2,1],[1,2],[1,3],[3,4],[0,5]]);
```





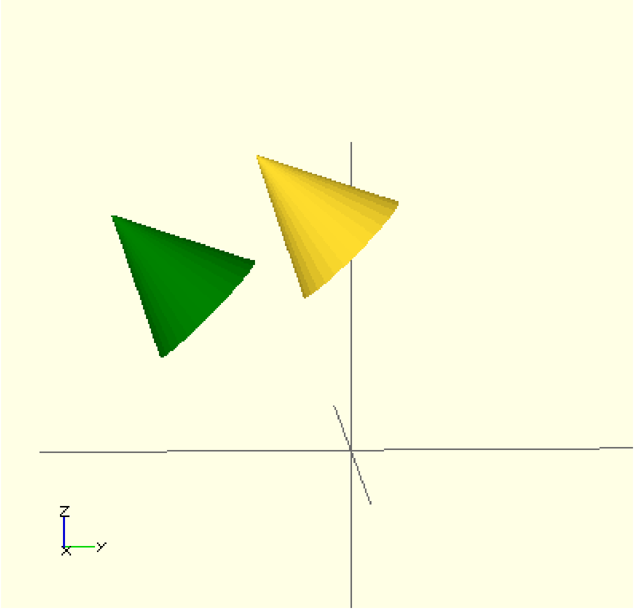
3D Primitives

- Sphere
  - `sphere(5);`
    `sphere(r=5);`
- Cube
  - `cube(5);`
  - `cube([4,8,16]);`
- Cylinder
  - `cylinder(20,10,5);`
    `cylinder(h = 20, r1 = 10, r2 = 5);`
  - `cylinder(h=20,r=10);`



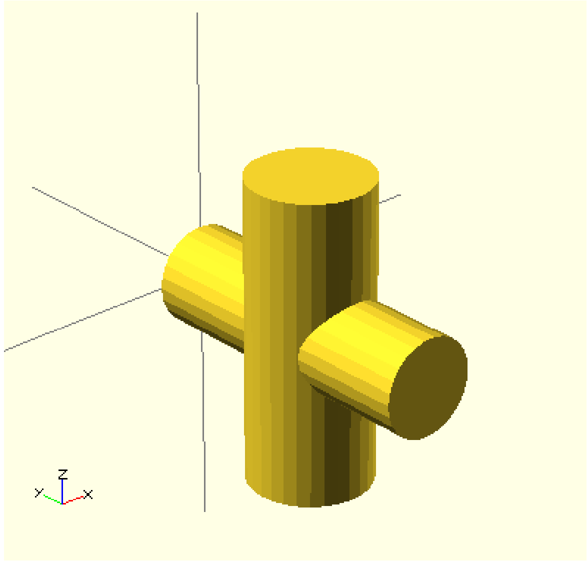
Transformations

- Translate
  - e.g., `translate([10,0,0]) sphere(5); // translate along x axis`
- Rotate
- Scale
- Order dependent
  - `Color("yellow") translate([0,0,10]) rotate([45,0,0]) cylinder([20,10,0]);`
  - `Color("green") rotate([45,0,0]) translate([0,0,10]) cylinder([20,10,0]);`



CSG

- Union
- Intersection
- Difference
- Example:

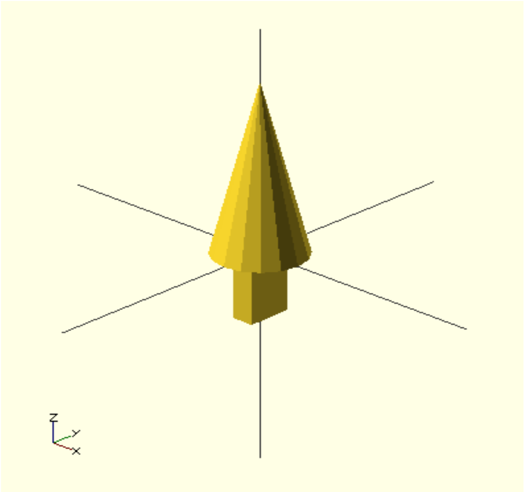



```
union()
{
        translate([0,-25,-25]) cylinder(50,10,10);
        rotate([90,0,0]) cylinder(50,8,8);
}
```

Module

- Procedures/Functions

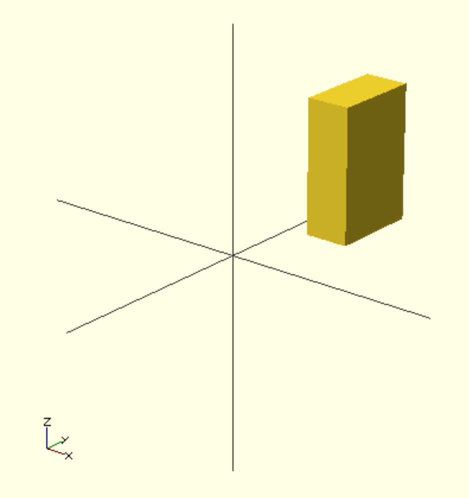
```
module leaves() { cylinder(20,5,0); }
module box() { cube([5,10,15]); }
module tree() {
      leaves();
      scale([0.5,0.5,0.5]) translate([-2.5,-5,-
      15]) box();
   }
tree();
```



Module

- Parameters

```
module box(w,l,h,tx,ty,tz){
        translate([tx,ty,tz])
        cube([w,l,h]);
}
box(5,10,15,10,0,5);
```

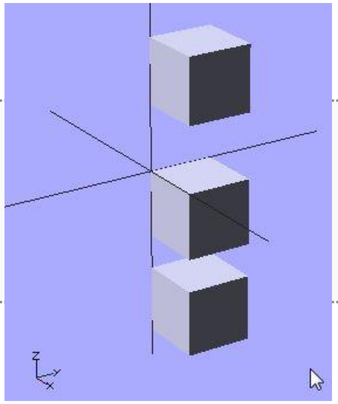


- Default values

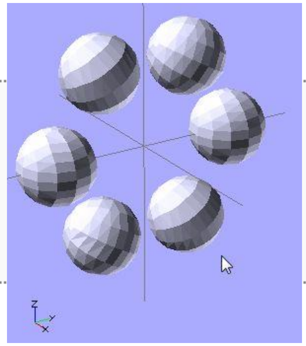
```
module box2(w=5,l=10,h=20){
        echo("w=", w, " l=", l, " h=", h);
        cube([w,l,h]);
}
box2();
```

Loops

```
for (loop_variable_name = range or vector) {
        ….
      }
```



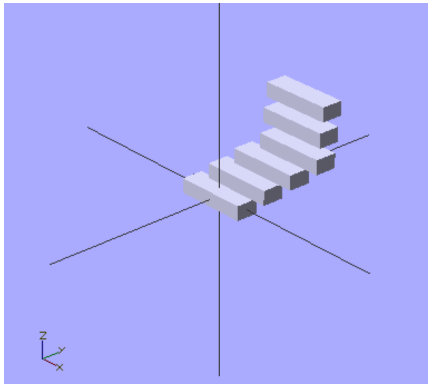
```
for ( z = [-1, 1, -2.5]) {
 translate( [0, 0, z] )
    cube(size = 1, center = false);
}
```



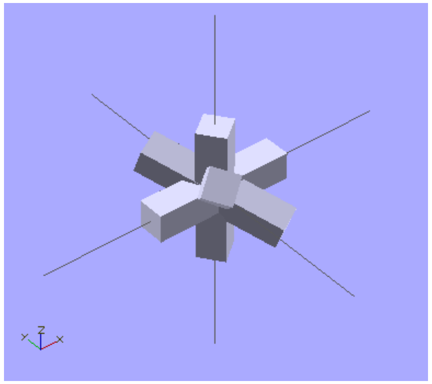
```
for ( i = [0:5] ) {
 rotate( i*360/6, [1, 0, 0])
  translate( [0, 10, 0] ) sphere(r = 1);
}
```

Loops

```
 for(i = [ [ 0,  0,  0],
           [10, 12, 10],
           [20, 24, 20],
           [30, 36, 30],
           [20, 48, 40],
           [10, 60, 50] ])
{
    translate(i)
    cube([50, 15, 10], center = true);
}
```



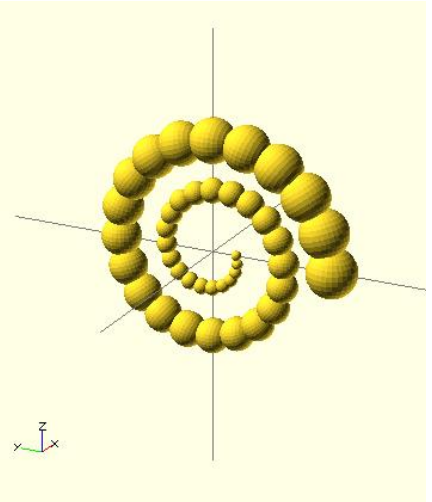

```
for(i = [ [  0,  0,   0],
          [ 10, 20, 300],
          [200, 40,  57],
          [ 20, 88,  57] ])
{
    rotate(i)
    cube([100, 20, 20], center = true);
}
```



Variables

- Assign() statement
  - In openscad, one can only assign variables at file top-level or module top-level
  - If you need it inside the for loop, you need to use assign(), e.g,:

```
for (i = [10:50])
    assign (angle = i*360/20, distance = i*10, r = i*2) {
        rotate(angle, [1, 0, 0])
            translate( [0, distance, 0] ) sphere(r = r);
    }
```



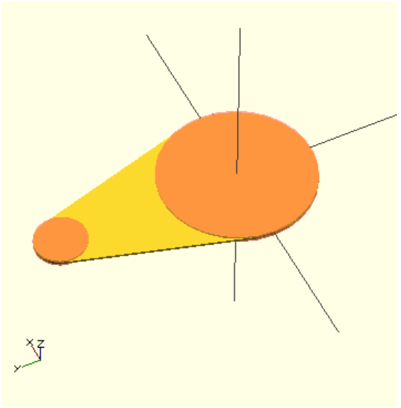
Conditionals

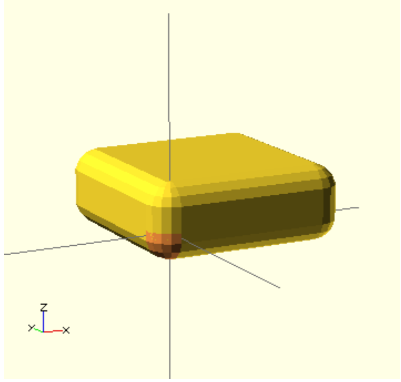
- If/else/else if
  - Syntax similar to C/C++

```
if (boolean_expression) { .... }
if (boolean_expression) { .... } else {.... }
if (boolean_expression) { .... } else if (boolean_expression) {.... }
if (boolean_expression) { .... } else if (boolean_expression) {.... } else {....}
```

Useful Functions

- mirror(): mirror the element on a plane through origin, argument is the normal vector of the plane, e.g., mirror([0,1,0]);
- hull(); create a convex hull from all objects that are inside, e.g., hull() {# translate([0,70,0]) circle(10); # circle(30); }
- minkowski(); takes one 2D shape and traces it around the edge of another 2D shape, e.g., minkowski() { cube([30,30,5]); # sphere(5);}





The Plan For Today

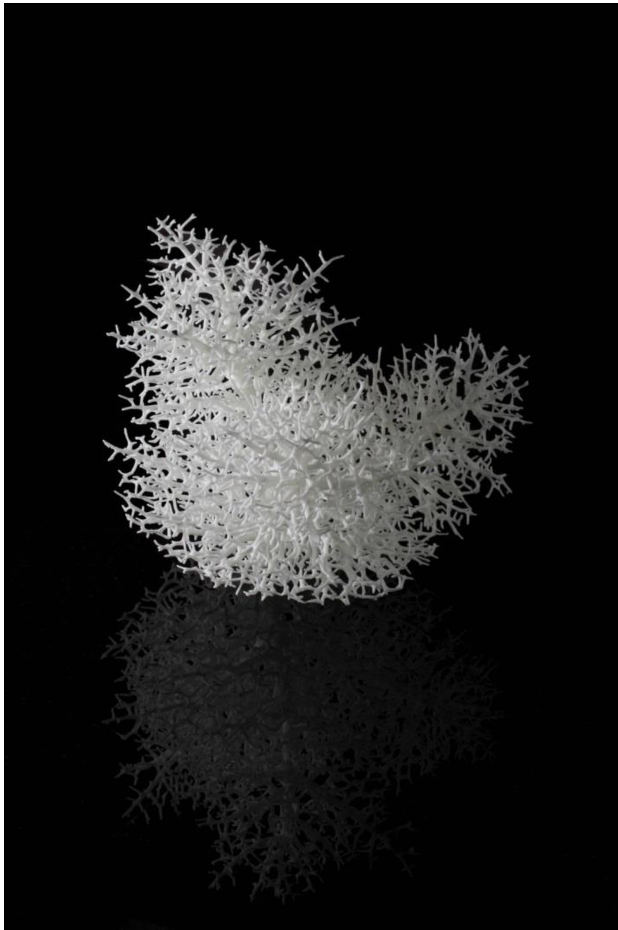
- Constructive Solid Geometry (CSG)
  - Parametric models from simple primitives
- Procedural Modeling

The Plan For Today

- Constructive Solid Geometry (CSG)
  - Parametric models from simple primitives
- Procedural Modeling

Procedural Modeling

- Goal:
  - Describe 3D models algorithmically
- Best for models resulting from ...
  - Repeating or similar structures
  - Random processes
- Advantages:
  - Automatic generation
  - Concise representation
  - Parameterized classes of models



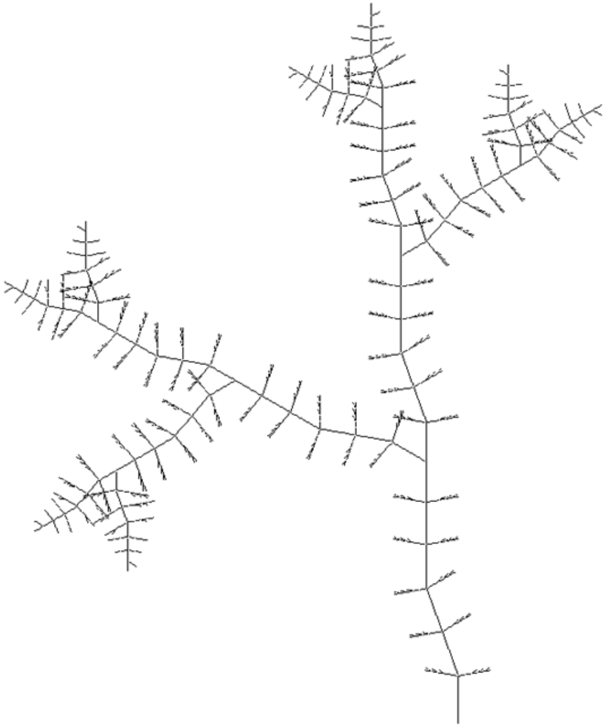
Formal Grammars and Languages

- A finite set of **nonterminal symbols: {S, A, B}**
- A finite set of **terminal symbols: {a, b}**
- A finite set of **production rules: S → AB; A → aBA**
- A **start symbol: S**

- Generates a set of finite-length sequences of symbols by recursively applying production rules starting with **S**

L-systems (Lindenmayer systems)

- A model of morphogenesis, based on formal grammars (set of rules and symbols)
- Introduced in 1968 by the Swedish biologist A. Lindenmayer
- Originally designed as a formal description of the development of simple multi-cellular organisms
- Later on, extended to describe higher plants and complex branching structures



L-system Example

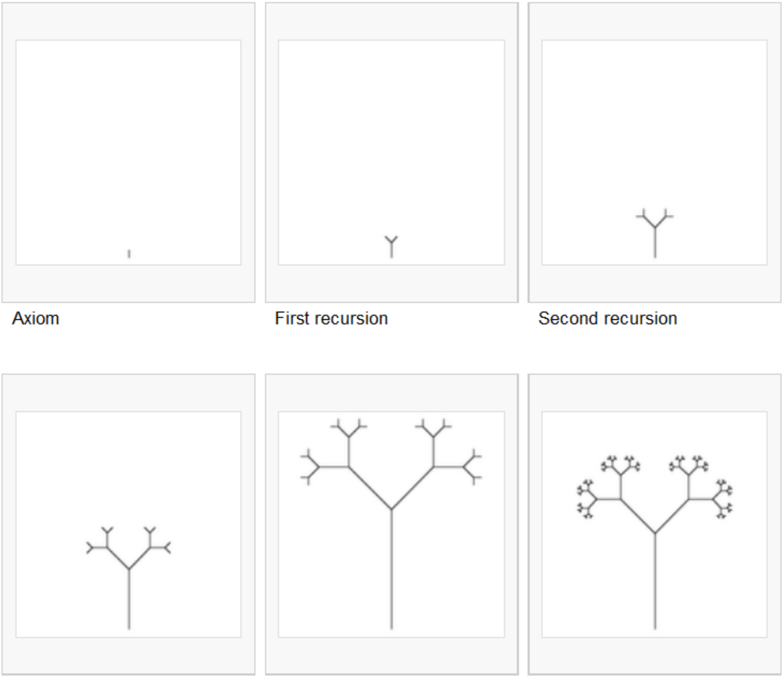
- **nonterminals** : 0, 1
- **terminals** : [,]
- **start** : 0
- **rules** : $(1 \rightarrow 11)$, $(0 \rightarrow 1[0]0)$

How does it work?

start:	0
1st recursion:	1[0]0
2nd recursion:	11[1[0]0]1[0]0
3rd recursion:	1111[11[1[0]0]1[0]0]11[1[0]0]1[0]0

L-system Example

- Visual representation: turtle graphics
  - 0: draw a line segment ending in a leaf
  - 1: draw a line segment
  - [: push position and angle, turn left 45 degrees
  -]: pop position and angle, turn right 45 degrees



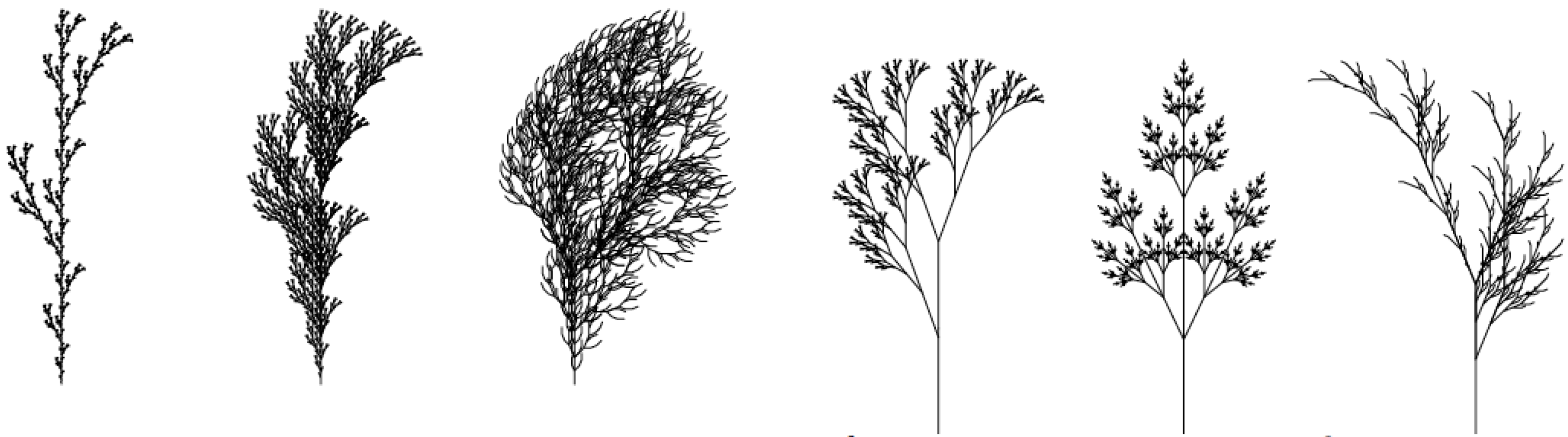
Axiom | First recursion | Second recursion | Third recursion | Fourth recursion | Seventh recursion, scaled down ten times

L-system Example 2: Fractal Plant

- **nonterminals** : X, F
- **terminals** : + - [ ]
- **start** : X
- **rules** : (X → F-[[X]+X]+F[+FX]-X), (F → FF)

L-Systems Examples

- Tree examples



a
n=5, δ=25.7°
F
F→F[+F]F[-F]F

b
n=5, δ=20°
F
F →F[+F]F[-F][F]

c
n=4, δ=22.5°
F
F→FF-[-F+F+F]+
[+F-F-F]

d
n=7, δ=20°
X
X →F[+X]F[-X]+X
F →FF

e
n=7, δ=25.7°
X
X→F[+X][-X]FX
F→FF

f
n=5, δ=22.5°
X
X→F-[[X]+X]+F[+FX]-X
F→FF

L-Systems Examples

Types of L-Systems

- *Deterministic*: If there is exactly one production for each symbol

  $0 \rightarrow 1[0]0$

- *Stochastic*: If there are several, and each is chosen with a certain probability during each iteration

  $0\ (0.5) \rightarrow 1[0]0$

  $0\ (0.5) \rightarrow 010$

Types of L-Systems

- *Context-free:* production rules refer only to an individual symbol
- *Context-sensitive:* the production rules apply to a particular symbol only if the symbol has certain neighbours

$$
\begin{aligned}
S &\rightarrow aSBC \\
S &\rightarrow aBC \\
CB &\rightarrow HB \\
HB &\rightarrow HC \\
HC &\rightarrow BC \\
aB &\rightarrow ab \\
bB &\rightarrow bb \\
bC &\rightarrow bc \\
cC &\rightarrow cc
\end{aligned}
$$

Types of L-Systems

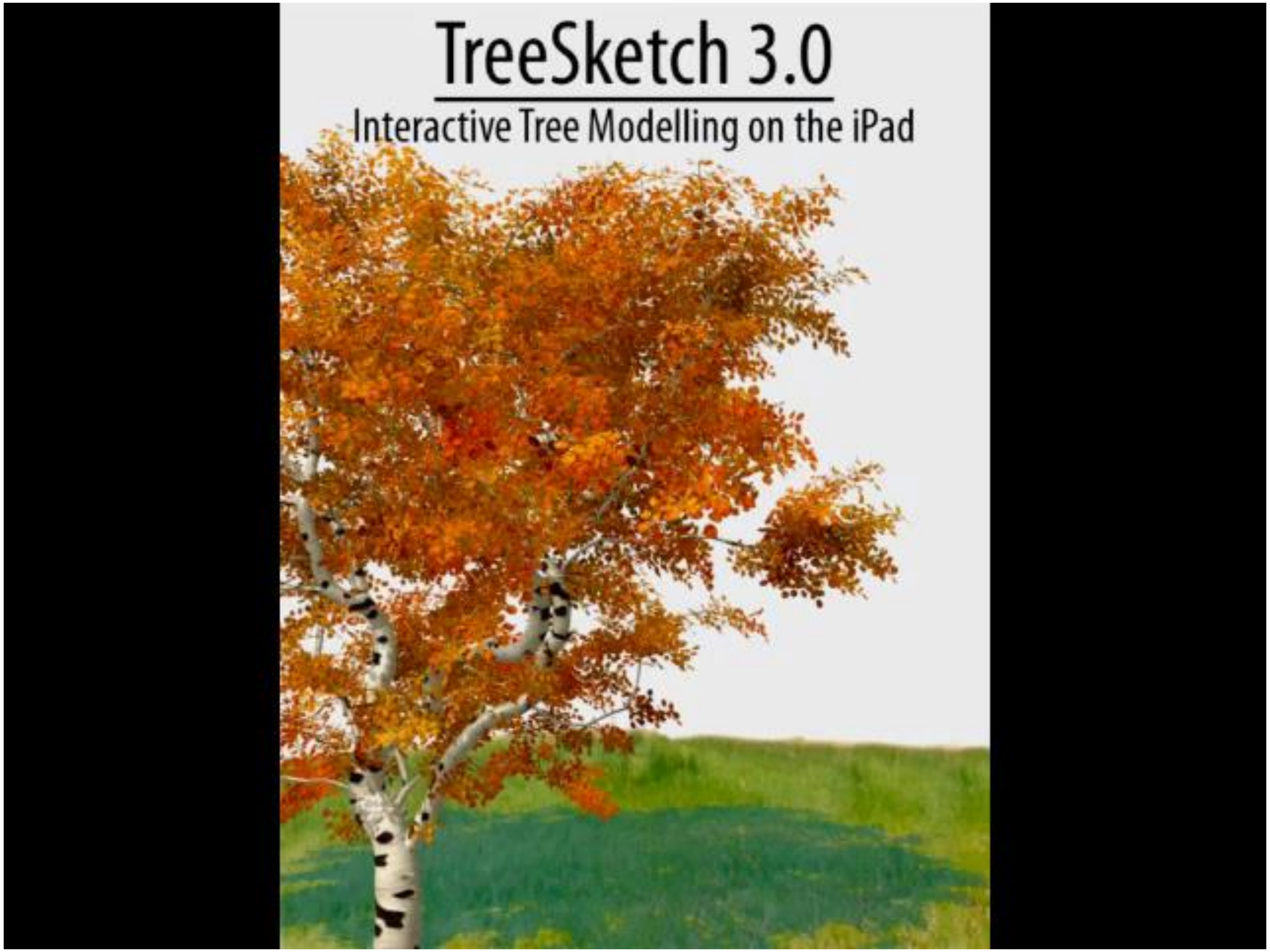
- *Nonparametric grammars:* no parameters associated with symbols
- *Parametric grammars:* symbols can have parameters
  - Parameters used in conditional rules
  - Production rules modify parameters

  - $A(x,y) \rightarrow A(1, y+1)B(x-2,3)$

Applications: Plant Modeling

- Algorithmic Botany @ the University of Calgary
  - Covers many variants of L-Systems, formal derivations, and exhaustive coverage of different plant types.
  - http://algorithmicbotany.org/papers
  - http://algorithmicbotany.org/virtual_laboratory/

TreeSketch: Interactive Tree Modeling



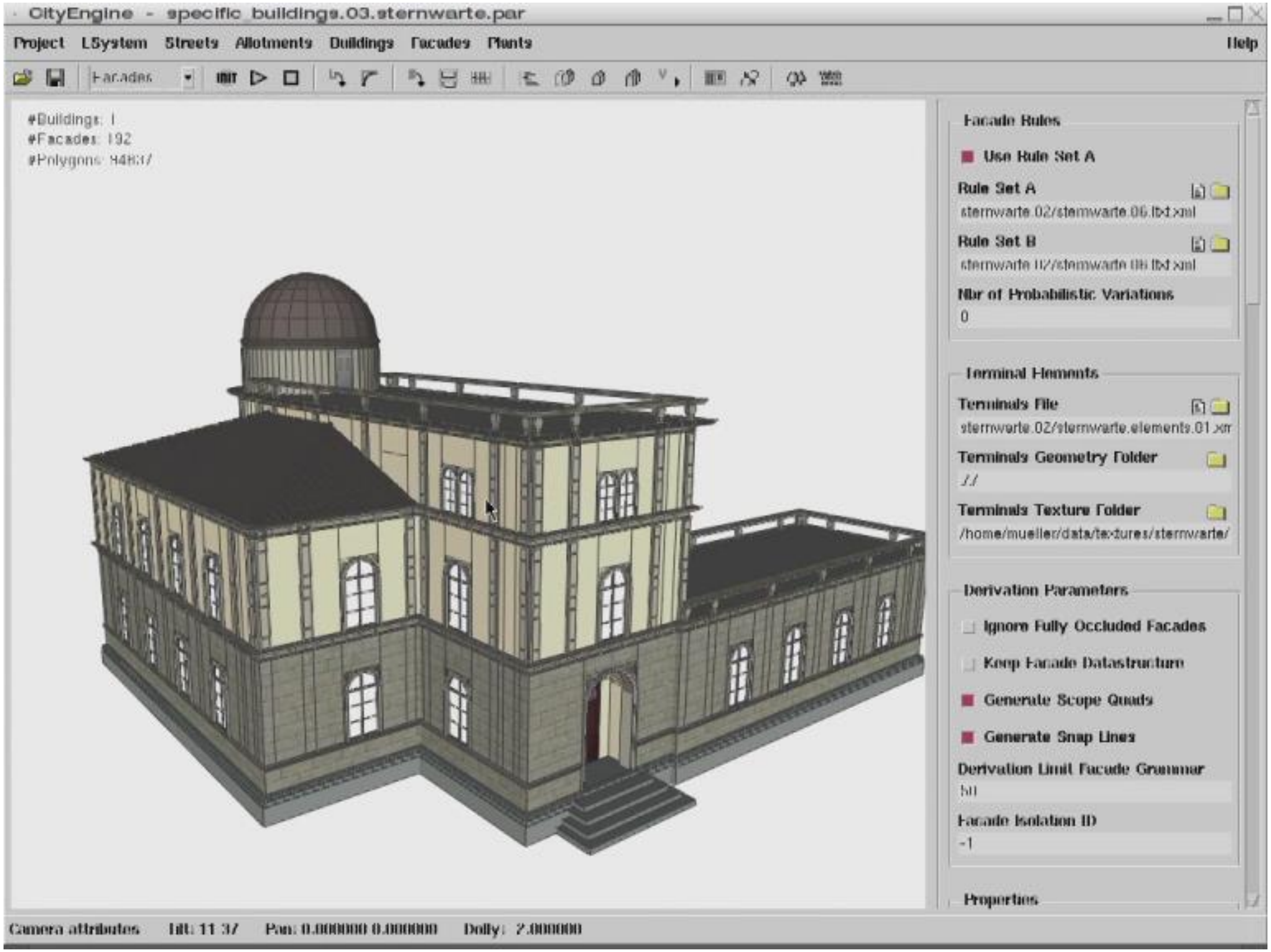
http://vimeo.com/68195050

Procedural Modeling of Buildings

- Pompeii

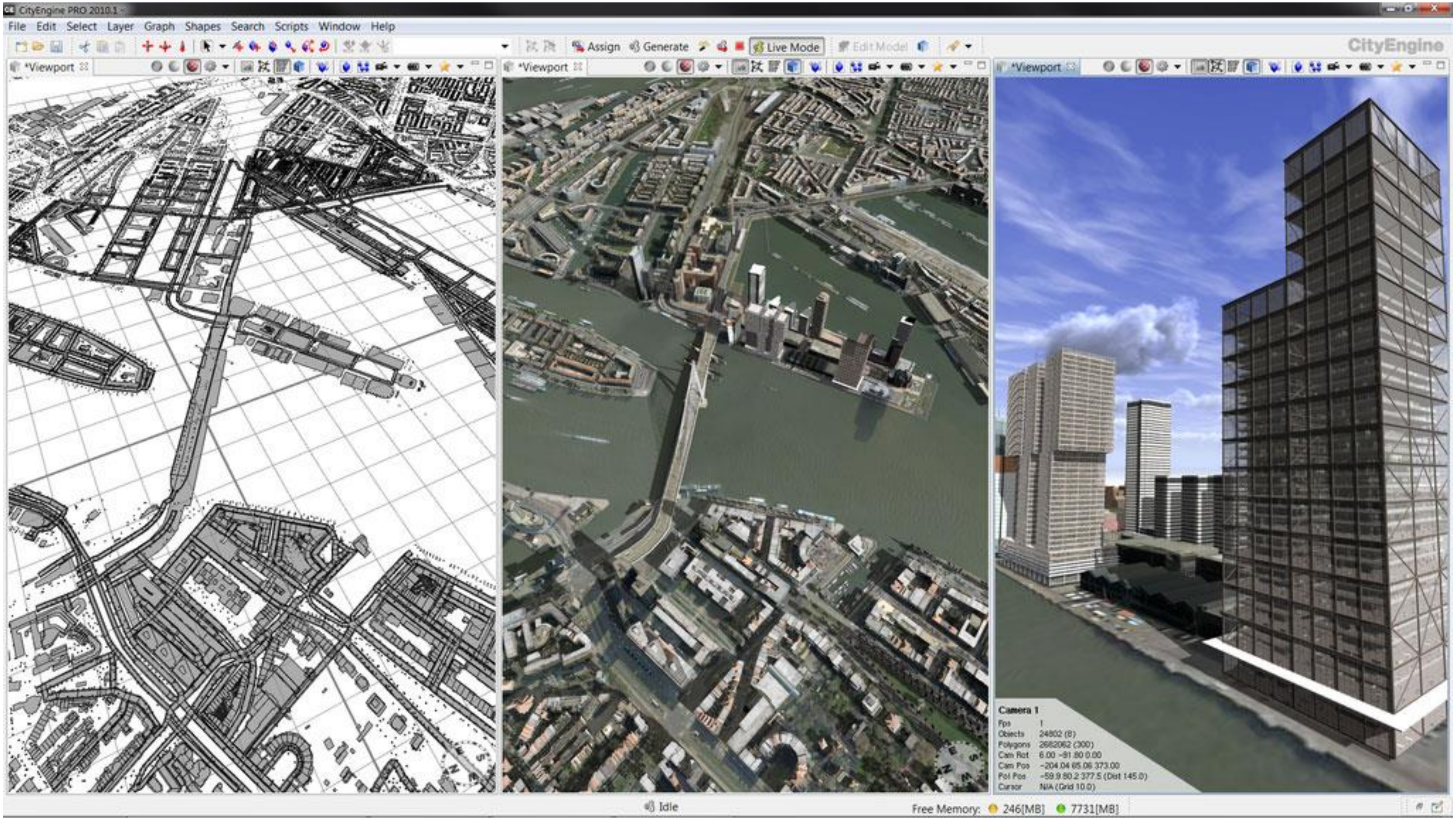
Procedural Modeling of Buildings / Müller et al, Siggraph 2006

Procedural Modeling of Buildings



Procedural Modeling of Buildings / Müller et al, Siggraph 2006

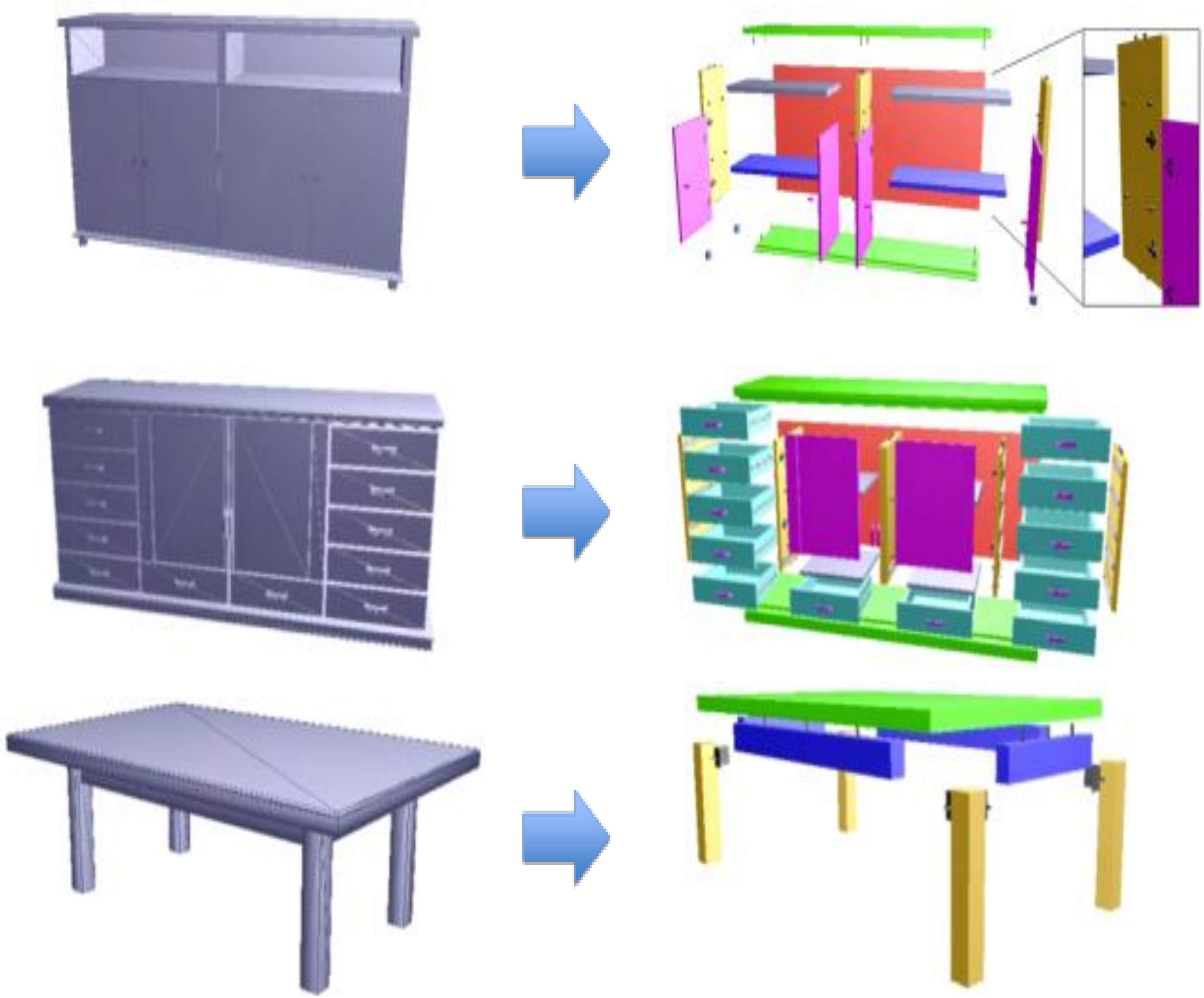
CityEngine



http://www.esri.com/software/cityengine/

Furniture Design

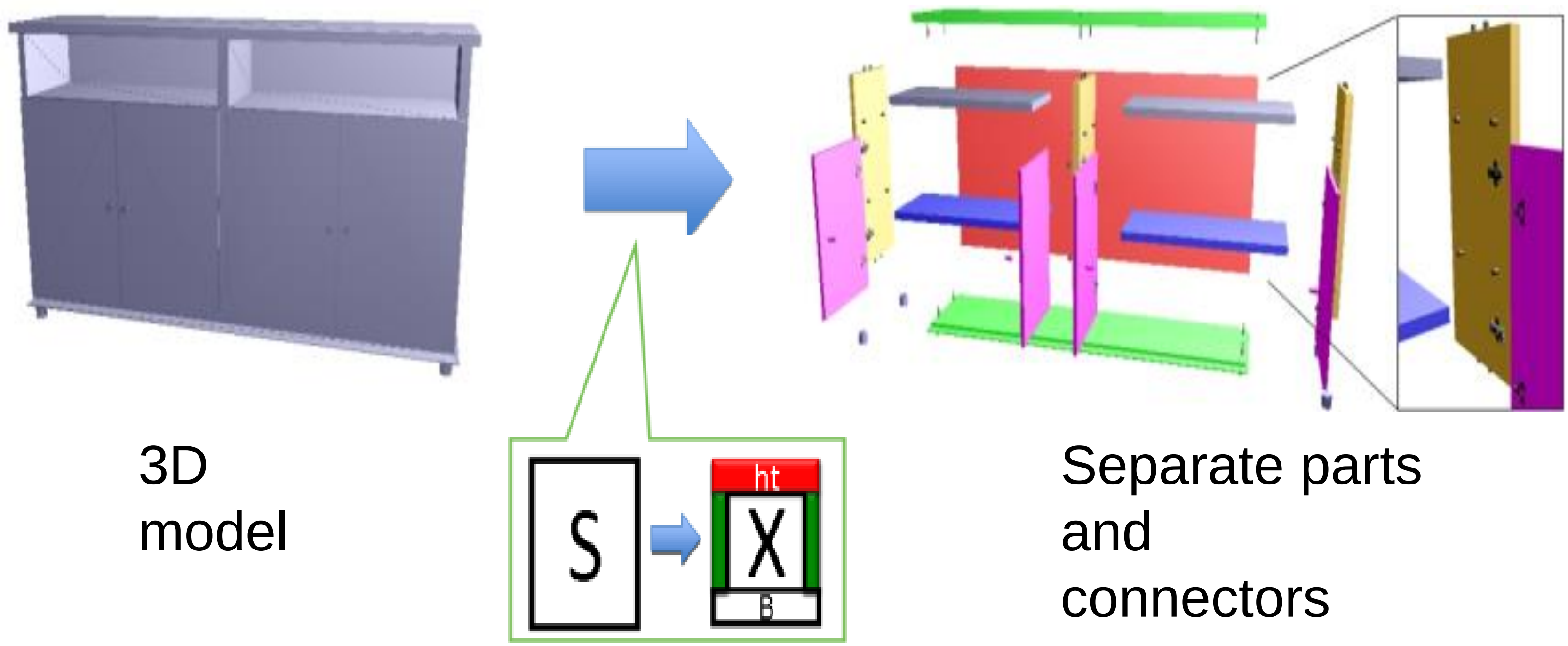
Input:
3D
model



Output:
Fabricatable
Parts and
Connectors

Converting 3D Furniture Models to Fabricable Parts and Connectors, Lau et al., Siggraph 2011

Approach



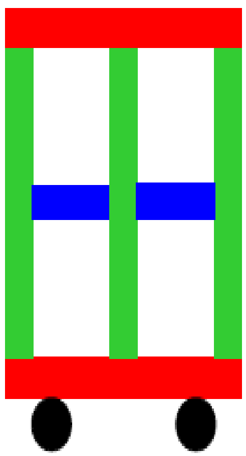
3D model

Separate parts and connectors

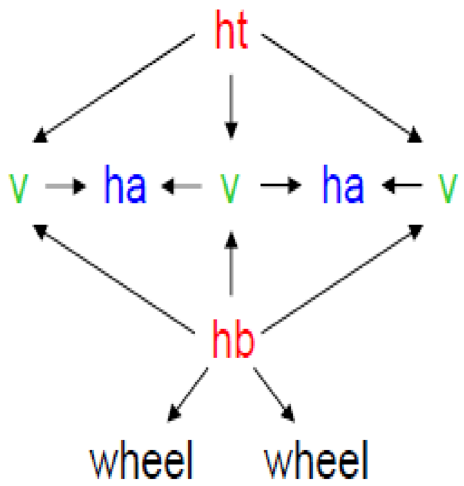
Formal grammar

Pre-defined formal grammar used to analyze structure of 3D models

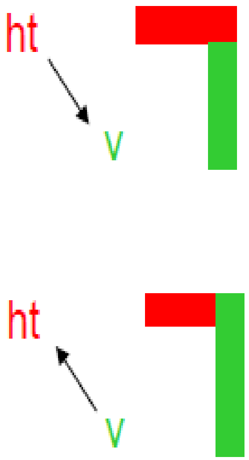
Example: 2D Cabinet



Example 2D Cabinet



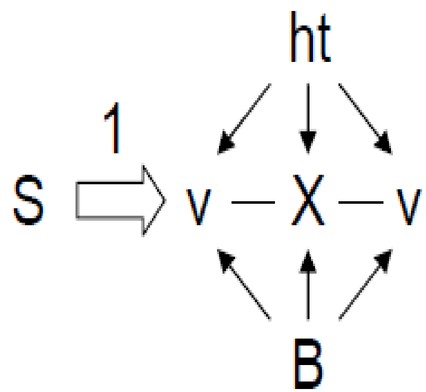
Corresponding Graph

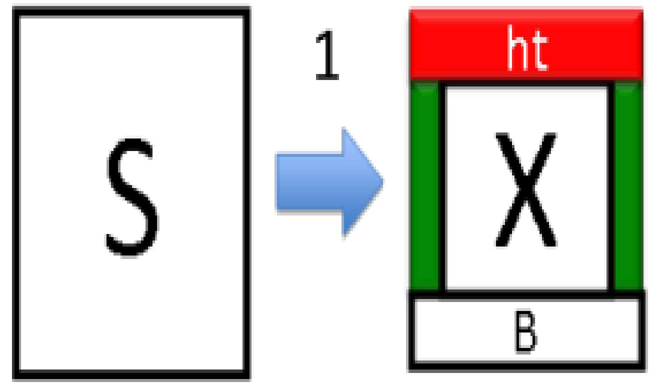


Positioning of Parts

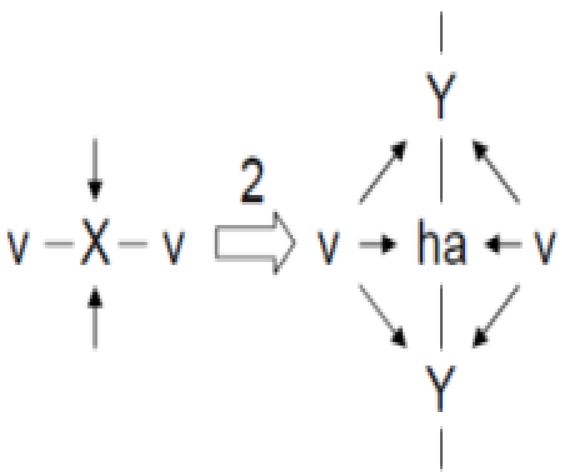
Examples of Production Rules

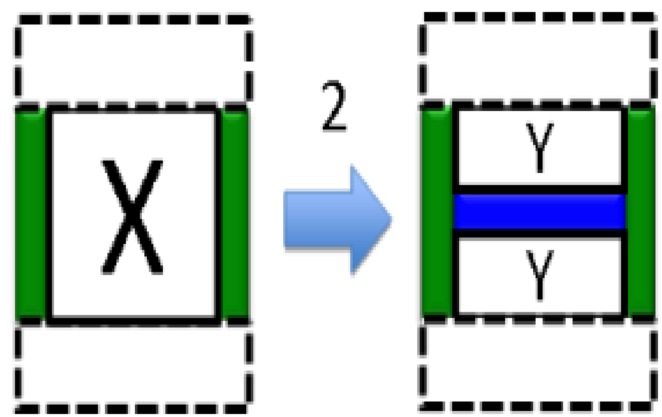
Production Rule 1





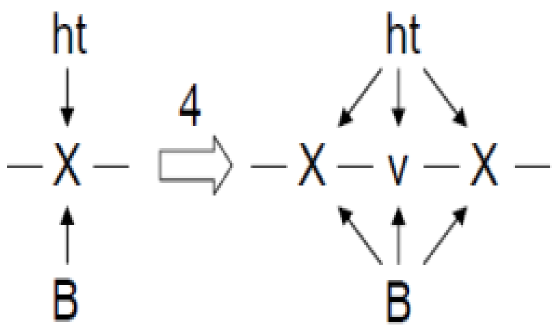
Production Rule 2

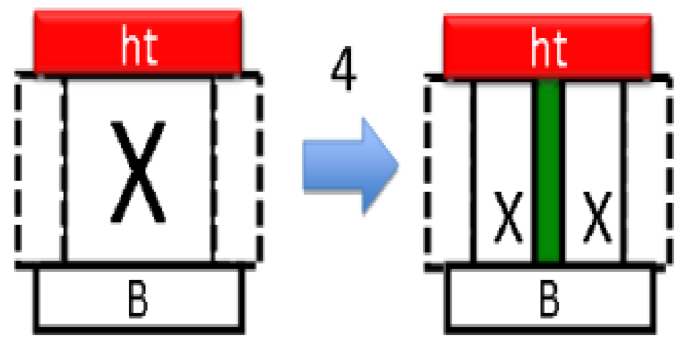




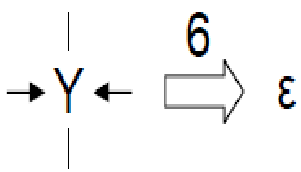
Examples of Production Rules

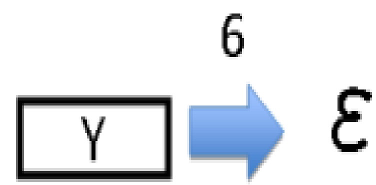
Production Rule 4



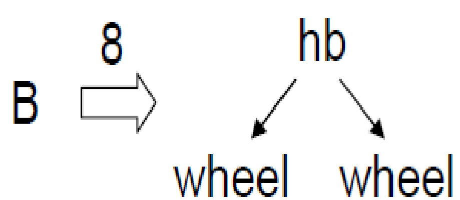


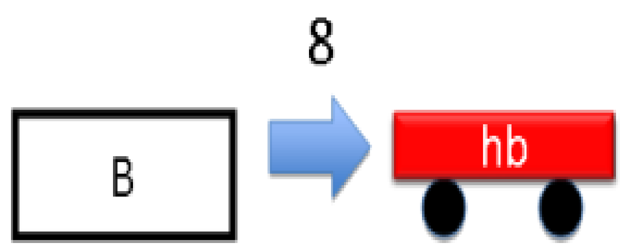
Production Rule 6



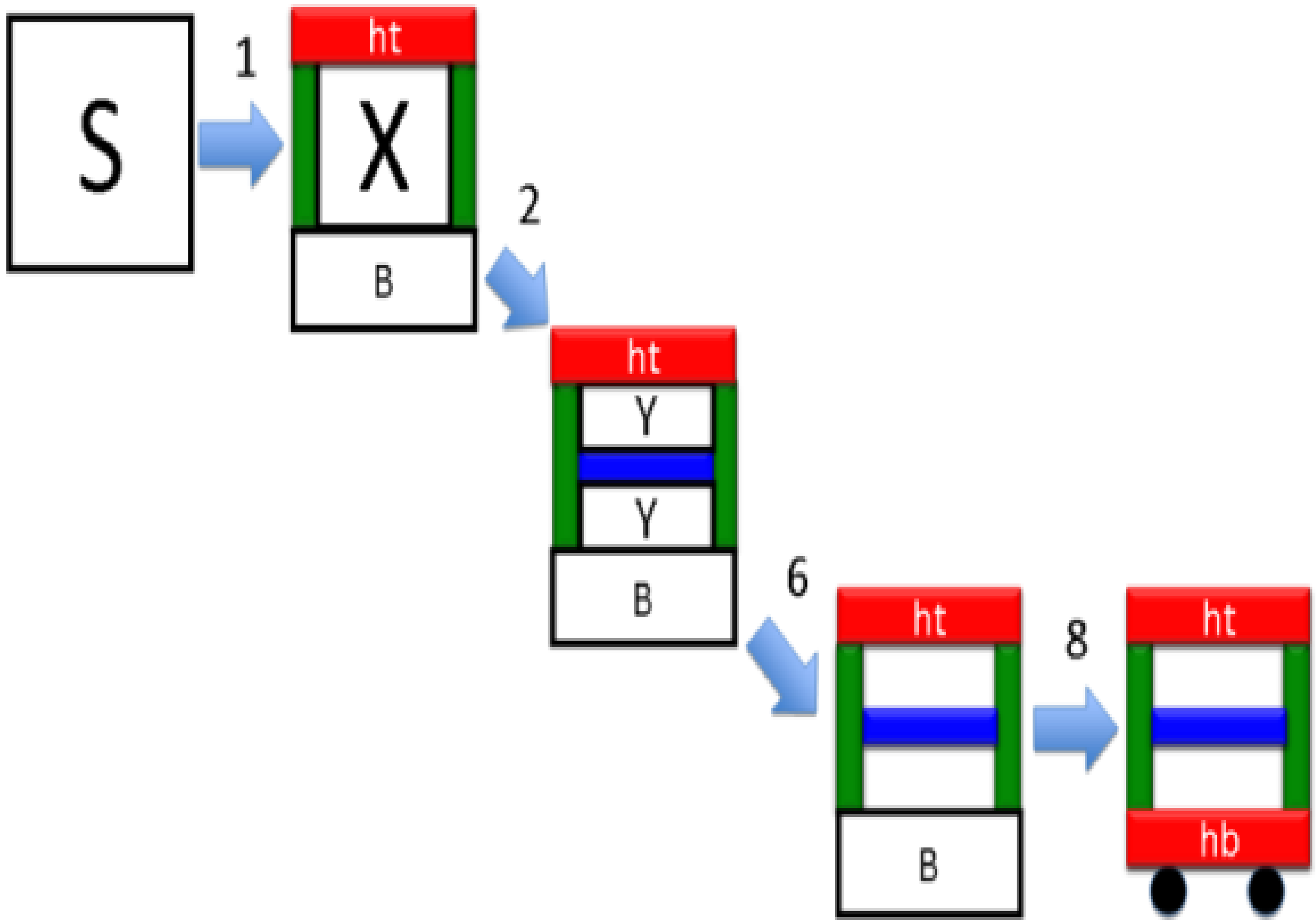


Production Rule 8

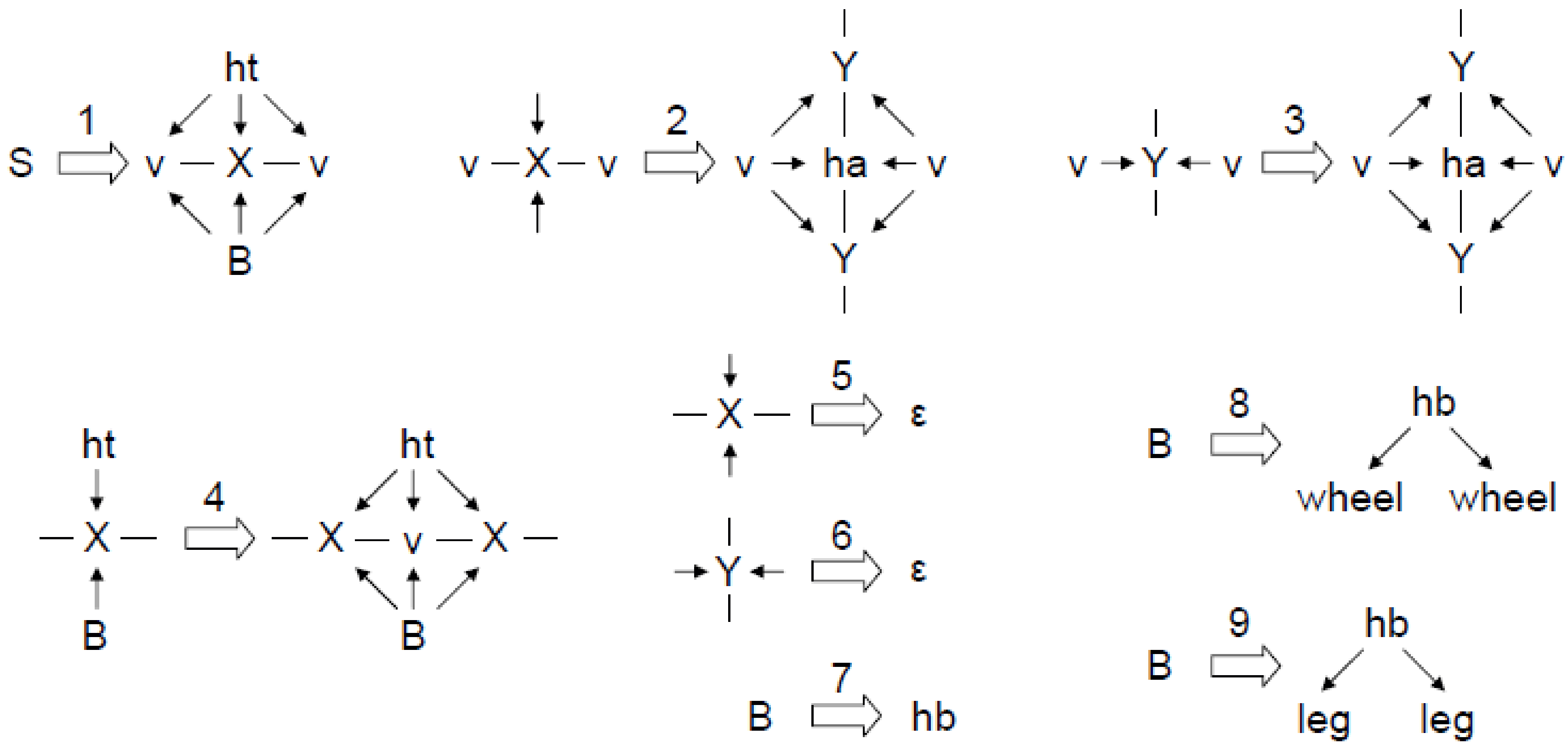




Sequence of Production Rules



All Production Rules

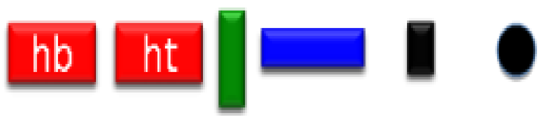


Formal Grammar for 2D Cabinets

$$N = \{\boxed{S}, \boxed{B}, \boxed{X}, \boxed{Y}\}$$

Non-terminal Symbols
- Collection of Parts

$$\Sigma = \{hb, ht, v, ha, leg, wheel\}$$



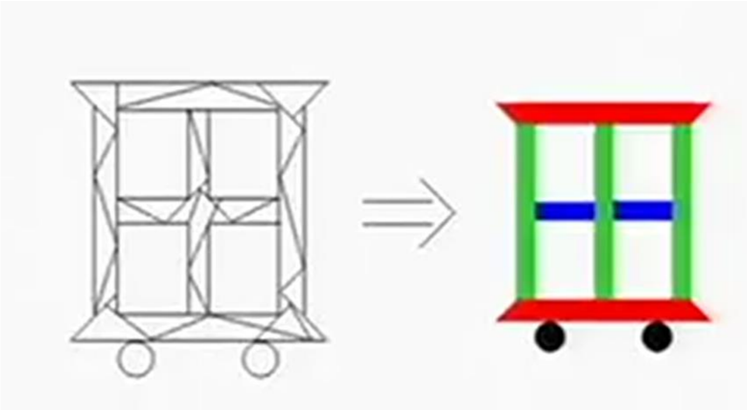
Terminal Symbols
- Separate Parts

P : Set of Production Rules

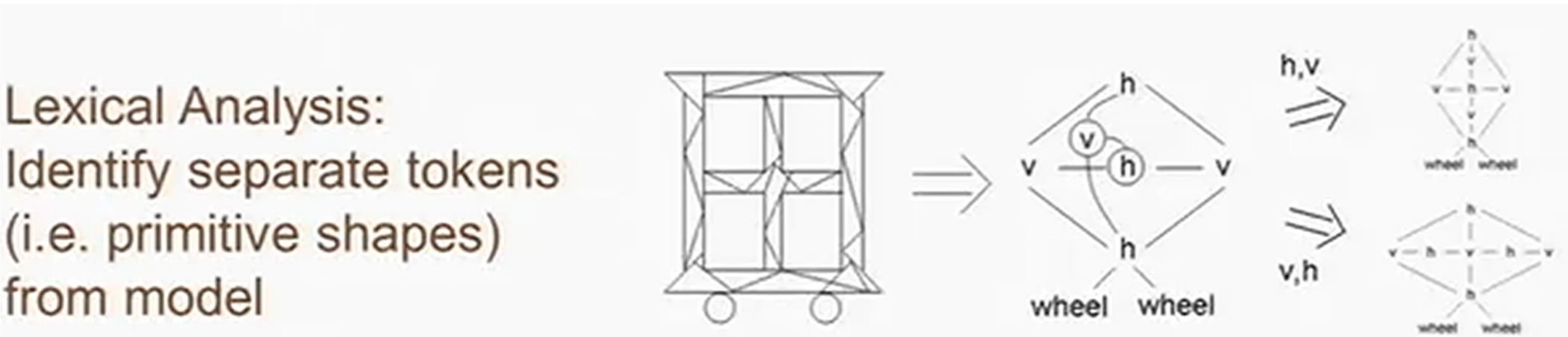
$\boxed{S}$: Start Symbol

The language specifies a directed graph which represents parts and connectors

Overview of algorithm



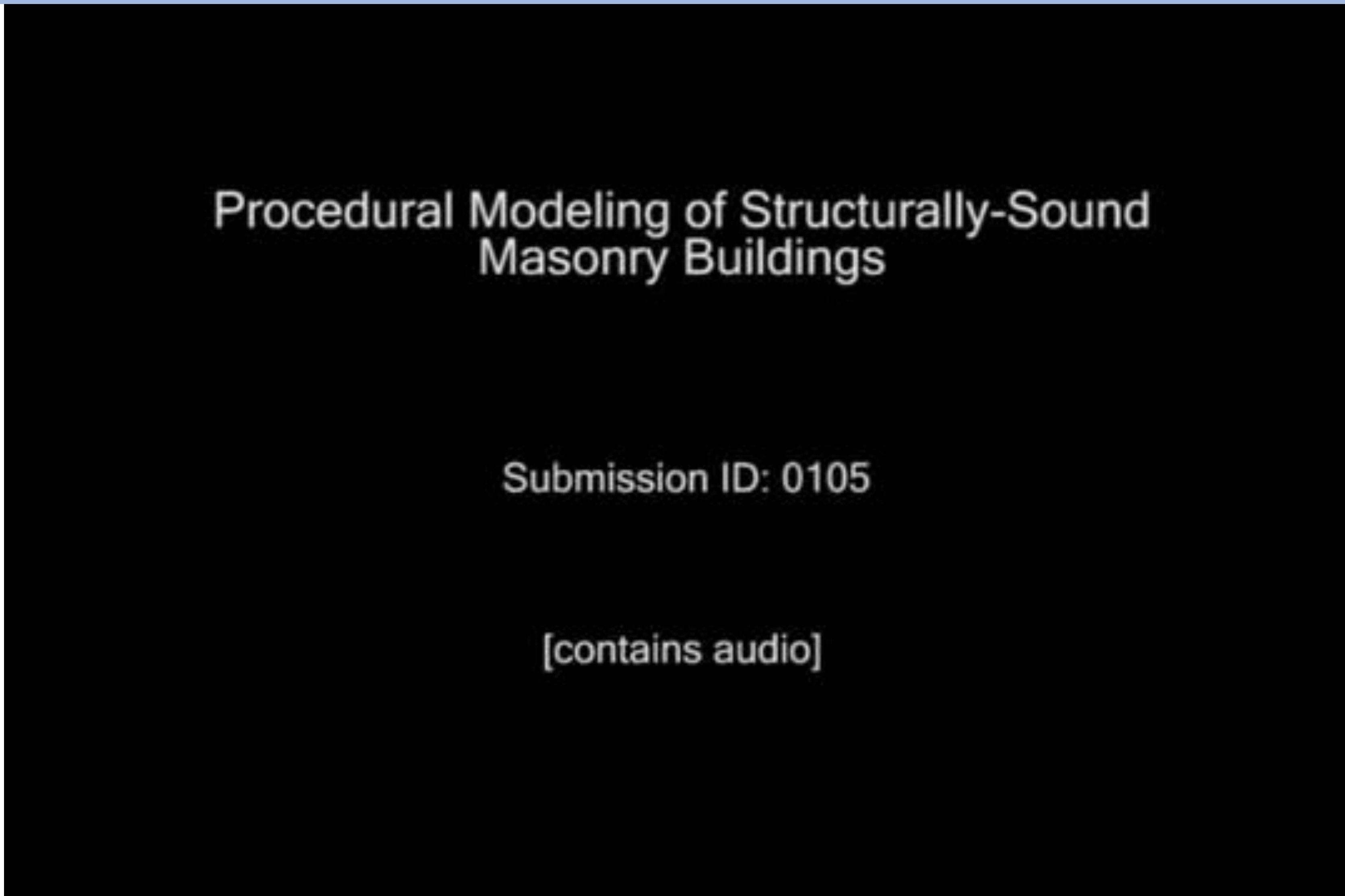
Overview of algorithm



Multiple valid options

Grammar-based Furniture Design

Procedural Modelling



That’s All For Today