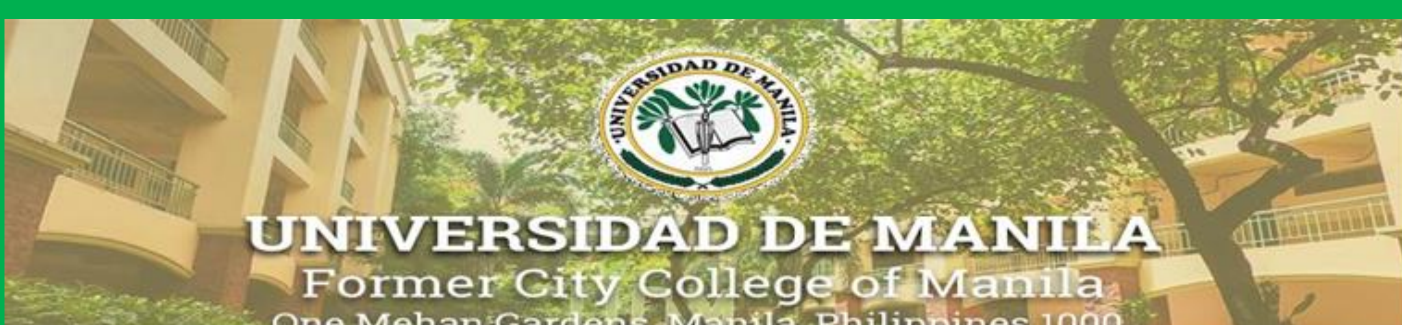
UNIVERSIDAD DE MANILA

Mathematics in the Modern World

2020

Learning Module



STUDENT
Name:
Student Number:
Program:
Year and Section:
Home Address:
Email address:
Contact Number:

PROFESSOR	
Name:	Prof: Rafael J. Eusebio Jr., LPT
Academic Department:	College of Education
Contact Details:	rafeusebio14@gmail.com / rafael_eusebio14@yahoo.com (+639)17 889 4470

i. Course Code	GECS004
II. Course Title	Mathematics in the Modern World
III. Module Number	
IV. Module Title	Nature of Mathematics
V. Overview of the Module	In this module, it will discuss and looking at patterns and regularities in the world, and how mathematics comes into play, both in nature and in human endeavors.
VI. Module Outcomes	As for the outcome of the module, you are expected to identify patterns in nature, articulate the importance of mathematics and express appreciation for mathematics as a human endeavor.
VII. General Instructions	*You must allow the necessary time to complete the lessons each week. If you choose not to complete the lesson using the schedule provided, you must understand that it is your full responsibility to complete them by the last day of completion. Time is of the essence.* *The module is designed to assess student understanding of the assigned lessons found within the associated content of the midterm and final period of the course. The assessment part of the module is composed of varied types of questions.* *You may see gamified assessment tools like crossword puzzles, scrambled puzzle figures, traditional assessments mechanical type of tests, and authentic assessments like reflection or simple research work. Pay attention to the answer to the assessment questions as you move through each lesson. After each module, you will be given a summative test. Your responses to the assessment parts of the module will be checked and recorded.* *Because the assessment questions are available within the whole completion period and because you can refer the answers to the questions within the content modules, the author-professor will not release the answers in the module. However, he is happy to*

	discuss the assessments with you during his consultation time, should you have any questions. *Good luck.* *You may not work collaboratively. This is independent work.* ***Added instruction with regards to the Progress Tracker*** **To track your progress in the completion of the module, each time you completed a task in each part of the lesson you will earn a badge. Now, to help you not to skip one of them, you are task to color the badges earned so that you can easily go back to the activities you remained idle.** **Budget your time wisely and effectively.** **Enjoy. Good luck!**

Module Progress Tracker

Badge Title	Progress	Badge Information
Motivated Badge	 ☆ ☆ ☆ ☆	**Completed the Motivational Activity**
Independent Reader Badge	 ☆ ☆ ☆ ☆	**Have read the entire Discussion.**
Practice Badge	 ☆ ☆ ☆ ☆	**Answered the Application part of the Lesson**
Reflection Badge	 ☆ ☆ ☆ ☆	**Have generalized the discussion through a reflection**
Quizzer Badge	 ☆ ☆ ☆ ☆	**Answered all the questions in the Assessment part**
Lesson Completer Badge	 ☆ ☆ ☆ ☆	**Completed all tasks in the entire Lesson.**

Lesson 1 | Patterns in Nature

Patterns exist in different variety of forms. The petals of a flower, arrangement of leaves reveals a sequential pattern. Natures are bounded by different colors and shapes – the rainbow mosaic of a butterfly's wings, the undulating ripples of a desert dune. But these miraculous creations not only delight the imagination, they also challenge our understanding. How do these patterns develop? What sorts of rules and guidelines, shape the patterns in the world around us?

Some patterns are molded with a strict regularity. At least superficially, the origin of regular patterns often seems easy to explain. Thousands of times over, the cells of a honeycomb repeat their hexagonal symmetry. The honeybee is a skilled and tireless artisan with an innate ability to measure the width and to gauge the thickness of the honeycomb it builds. Although the workings of an insect's mind may baffle biologists, the regularity of the honeycomb attests to the honey bee's remarkable architectural abilities.

Lesson Objectives | At the end of the lesson, you will be able to:

1. Identify patterns in nature and regularities in the world;
2. Explain the importance of mathematics in one's life; and
3. Express appreciation for mathematics as a human endeavor.

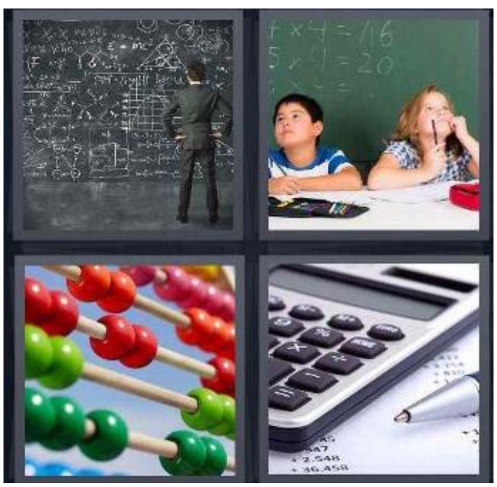
Motivation | "4 PICS 1 WORD" Activity

Guess the word that represents by the given pictures

1. | | | | | | |

2.

3.



4.

5.

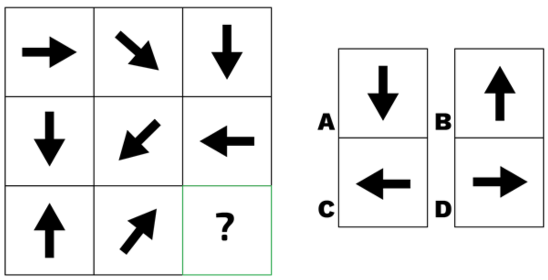
Congratulations! You earned a Motivated Badge

Discussion	Let us Begin!

A pattern is something which helps us anticipate what we might see or expect to happen next. It may also help us know what may have come before or what we are seeing currently. There are four types of patterns; (1) logic patterns, (2) number patterns, (3) geometric patterns and (4) word patterns.

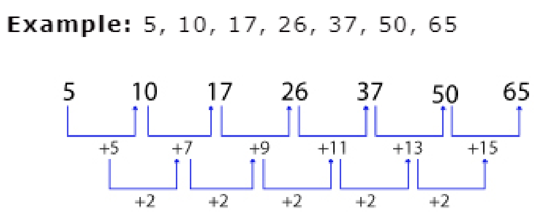
1. **Logic pattern** is the ability to discover meaningful patterns in strange and unpredictable situations. When you enter a strange space, like a new job, you spontaneously search for patterns to influence how you think and act and speak in the new space.

   Example of logical pattern:



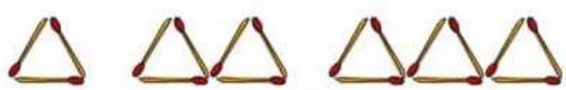
2. **Number pattern** is a sequence of number that are formed in accordance with a definite rule. We can often describe number patterns in more than one way. To illustrate this, consider the following sequence of numbers {1, 3, 5, 7, 9, …}. Clearly, the first term of this number pattern is 1; and the terms after the first term are obtained by adding 2 to the previous term. We can also describe this number pattern as a set of odd numbers.

   Example of number pattern:



3. **Geometric pattern** is a pattern that represented by geometrical figures such as polygons and isometric shapes.

   Example of geometric patterns:



No. of triangles	1	2	3	4	5	6
No. of matches	3	6	9	12	15	18

4. **Word pattern** are represented by jumbled words and analysed the hidden logic in it.

   Example of word pattern:

Patterns are part of our everyday life and are visible in shapes, color, number, and object repetition. Below are some of the patterns in nature;

1. **Symmetry**

   Symmetry is pervasive in living things. Animals mainly have bilateral or mirror symmetry, as do the leaves of plants and some flowers such as orchids. Plants often have radial or rotational symmetry, as do many flowers and some groups of animals such as sea anemones. Fivefold symmetry is found in the echinoderms, the group that includes starfish, sea urchins, and sea lilies.

   Examples:


Snowflakes


Tiger


Starfish

2. Trees and Fractals

The branching pattern of trees was described in the Italian Renaissance by Leonardo da Vinci. He stated that:

All the branches of a tree at every stage of its height when put together are equal in thickness to the trunk.

Fractals are infinitely self-similar, iterated mathematical constructs having fractal dimension. Infinite iteration is not possible in nature so all 'fractal' patterns are only approximate.

Examples:


Lindenmayer system


Angelica flower head


Trees: Lichtenberg

3. Spirals

Spirals are common in plants and in some animals, notably mollusks. For example, in the nautilus, a cephalopod mollusk, each chamber of its shell is an approximate copy of the next one, scaled by a constant factor and arranged in a logarithmic spiral. Given a modern understanding of fractals, a growth spiral can be seen as a special case of self-similarity.

Examples:


Nautilus shell


Sunflower

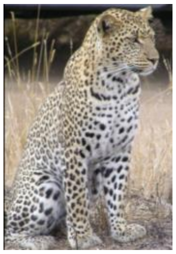

Aloe polyphylla

4. **Spots and Stripes**

Leopards and ladybirds are spotted; angelfish and zebras are striped. These patterns have an evolutionary explanation: they have functions which increase the chances that the offspring of the patterned animal will survive to reproduce. One function of animal patterns is camouflage; for instance, a leopard that is harder to see catches more prey.

Examples:


Zebra


Leopard


Angelfish

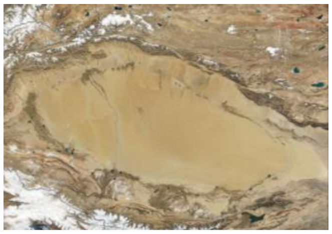
5. **Waves and Dunes**

Waves are disturbances that carry energy as they move. Mechanical waves propagate through a medium – air or water, making it oscillate as they pass by. Wind waves are sea surface waves that create the characteristic chaotic pattern of any large body of water, though their statistical behavior can be predicted with wind wave models. As waves in water or wind pass over sand, they create patterns of ripples. When winds blow over large bodies of sand, they create dunes, sometimes in extensive dune fields as in the Taklamakan desert. Dunes may form a range of patterns including crescents, very long straight lines, stars, domes, parabolas, and longitudinal or seif ('sword') shapes.

Examples:


Waves

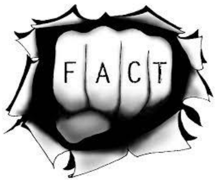
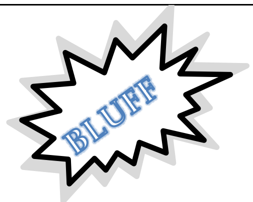
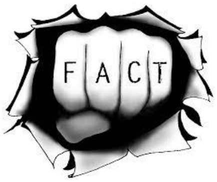
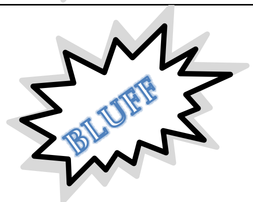
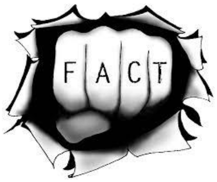
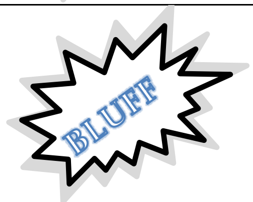
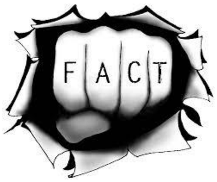
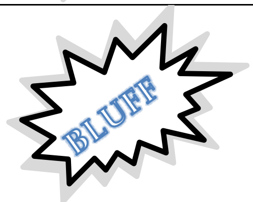

Wind ripples

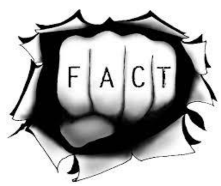
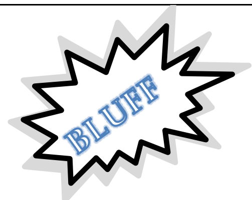

Dunes

Congratulations! You have earned an Independent Reading Badge

Application | Fact or Bluff!

Test your understanding. Each number has 2 icons, color the **fact** icon if the statement is correct, and the **bluff** icon if otherwise. (10 points)

		1. Geometric pattern is a sequence of number that are formed in accordance with a definite rule.
		2. A pattern is something which helps us anticipate what we might see or expect to happen next.
		3. Patterns can't be seen in nature.
		4. Symmetry patterns can be seen in starfish.

		5. Logic pattern is the ability to discover meaningful patterns in strange and unpredictable situations.

Congratulations! You have earned a **Practice Badge**

Generalization | Reflect the use of it!

In an essay no more than 100 words, how do you think learning mathematics and patterns in general are important in your daily lives?

____

____

____

____

____

____

____

____

Congratulations! You have earned a **Reflection Badge**

Assessment: It is your turn!

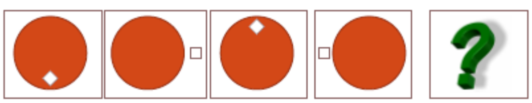
Evaluate each of the following:

1. Guess what's Next..

$$1 * 1 = 1$$
$$11 * 11 = 121$$
$$111 * 111 = 12{,}321$$
$$1{,}111 * 1{,}111 = 1{,}234{,}321$$
$$11{,}111 * 11{,}111 = 123{,}454{,}321$$
$$111{,}111 * 111{,}111 = \ ????$$

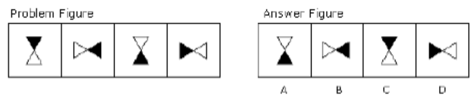
NEXTSTEP

2. What's the next shape?



3. **What's the next shape?**

4. Which symbol in the Answer Figure completes the sequence in the Problem Figure ?



5. What's the next number?

1. 2,4,6,8,10, ______
2. 1,3,5,7,9,11, ________
3. 1,3,7,15,31,________
4. 448,224,112,56,28,_______

Congratulations! You have earned a Quizzer Badge

List of References:

https://en.wikipedia.org/wiki/Patterns_in_nature#:~:text=Patterns%20in%20nature%20are%20visible,%2C%20tessellations%2C%20cracks%20and%20stripes.

Icons and Images

https://www.google.com/search?q=fact+fist&tbm=isch&ved=2ahUKEwj5psnQ45TqAhUnw4sBHVqRCAkQ2-cCegQIABAA&oq=fact+fist&gs_lcp=CgNpbWcQAzoCCAA6BggAEAoQGDoECAAQGDoGCAAQCBAeUJjkBFj6mgVgkpwFaAVwAHgCgAGLA4gBnA-SAQcyLjYuMS4ymAEAoAEBqgELZ3dzLXdpei1pbWc&sclient=img&ei=d03wXrmPNaeGr7wP2qKiSA#imgrc=_nh5wNTYRGcWBM

https://www.freepik.com/free-vector/collection-flat-colored-badge_861042.htm

https://www.freepik.com/premium-vector/gold-medal-golden-1st-place-badge-sport-game-golden-challenge-award-red-ribbon-realistic_6373639.htm

https://www.vectorstock.com/royalty-free-vector/modern-flat-design-badge-icon-vector-10839496

https://4pics1word-answers.com/level-3090/

https://www.slideshare.net/JecelleMarlon/4-pics-1-word-environment

Congratulations! You have earned Lesson Completer Badge.

Lesson 2 | Fibonacci Sequence and Golden Ratio

Pattern in nature exist with number sequence. It is very essential to understand that nature is not just an appreciation of the eye, but if you more closely understand the science you will also understand mathematics. Number of petals in a flower, rotation of leaves in a plant, number of seeds in a sunflower and even a spiral nautilus shells exhibit number sequence pattern. That is Fibonacci's number or Fibonacci sequence.

There are lots of available names for Fibonacci, some are Leonardo of Pisa, Leonardo Pisano and Leonardo Bonacci. All of them are considered name for Fibonacci. But who is Fibonacci? He was a mathematician, born in Pisa, Italy in 1175 AD, wrote different books (with essential contribution to mathematics) and an inventor of the famous sequence called "The Fibonnacci Sequence".

Lesson Objectives | At the end of the lesson, you will be able to:

1. Familiarized the Fibonacci's number;
2. Discuss the patterns in nature using Fibonacci's number; and
3. Discuss the golden ratio and its applications.

Motivation | "Maze O' Runner" Activity

Situation: Before proceeding to the World of Learning, the traveler needs to pass through Crossword Maze.

Task: Recall some terms that we discussed last topic. (15 points)

Clues:

Down:

1. It is something which helps us anticipate what we might see or expect to happen next.

4. Quality of being made up of exactly similar parts facing each other. It can be seen in starfish

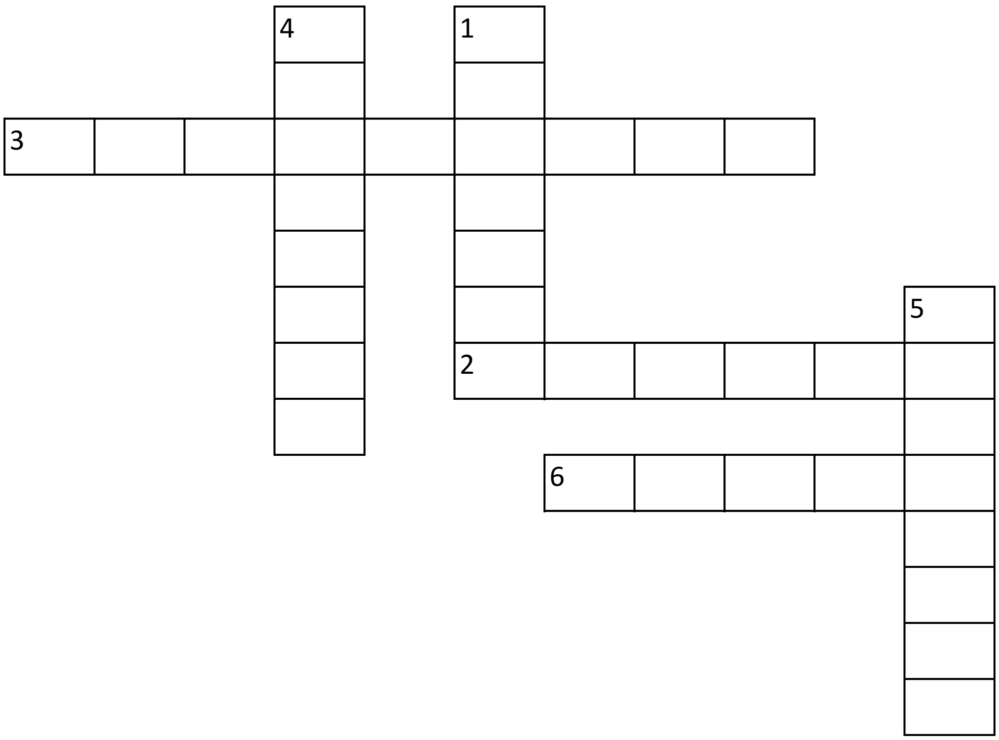
5. An object or quantity that displays self-similarity

Across:

2. Pattern that uses sequence of number that are formed in accordance with a definite rule

3. Pattern that represented by geometrical figures such as polygons and isometric shapes
6. Ability to discover meaningful patterns in strange and unpredictable situations

Crossword Maze



.

Welcome to the World of Learning

Congratulations! You have earned a Motivated Badge

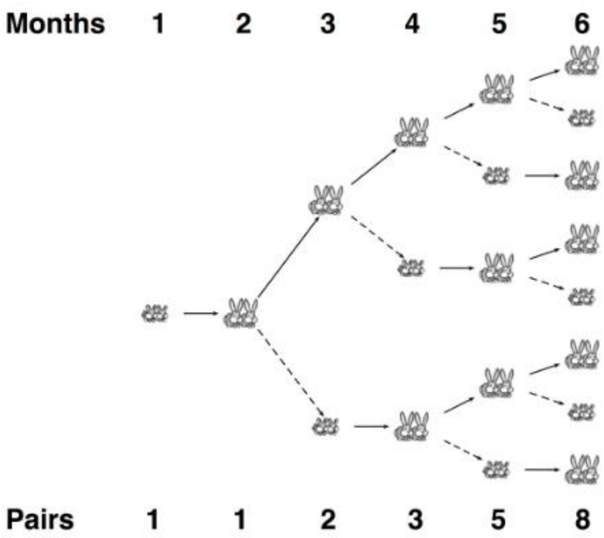
Discussion | "Learn More from Me"

I. Fibonacci Sequence

One of the prominent mathematician of medieval Europe is Leonardo of Pisa, known as Fibonacci. Below is the problem created by Fibonacci that concerns the birth of rabbits.

At the beginning of a month, you are given a pair of newborn rabbits. After a month, the rabbits have produced no offspring; however, every month thereafter, the pair of rabbits produces another pair of rabbits. The offspring reproduce in exactly the same manner. If none of the rabbits dies, how many pairs of rabbits will there be at the start of each succeeding month.

We will use Fibonacci sequence to solve the problem above. Figure below shows the numbers of pairs of rabbits on the first day of each of the first six months. The larger rabbits represents the matured rabbits that produce another pair of rabbits each month. The numbers 1, 1, 2, 3, 5, 5, 8 are the first six terms of the Fibonacci sequence.



Fibonacci discovered that the number of pairs of rabbits for any month after the first two months can be determined by adding the numbers of pairs of rabbits in each of the previous months, For instance, the number of pairs of rabbits at the start of the sixth month is 3 + 5 = 8.

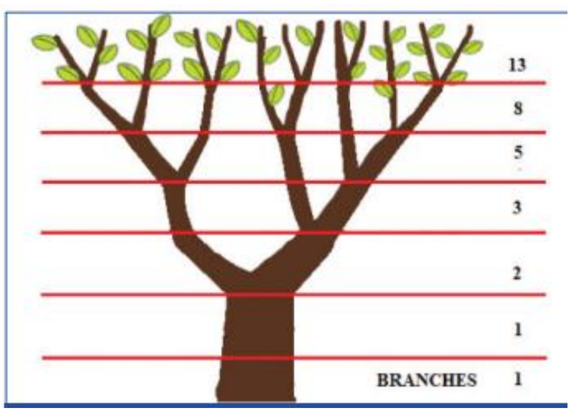
A recursive definition for a sequence is one in which each successive term of the sequence is defined by using some of the preceding terms. If we use the mathematical notation F_n to represent the nth Fibonacci number, then the numbers in the Fibonacci sequence are given by the following recursive definition:

$$F_1 = 1,\ F_2 = 1,\ F_n = F_{n-1} + F_{n-2} \text{ for } n \geq 3.$$

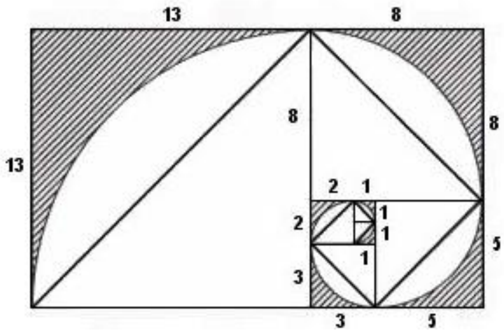
The Fibonacci sequence have numerous naturally occurring applications, ranging from the very basic to the complex geometric shapes.

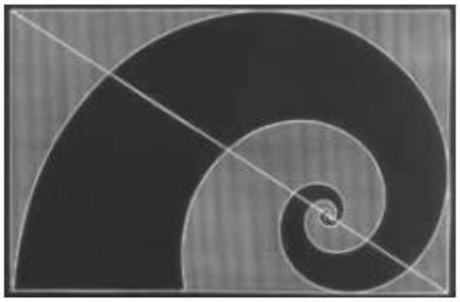
Example 1: The number of petals on a flower tend to be a Fibonacci number.

Example 2: Branching plants always branch off into groups of Fibonacci numbers.



Fibonacci numbers have geometric application in nature as well. The most prominent of these is the Fibonacci spiral on a nautilus shell.





Example 1: Find the sixth and seventh Fibonacci numbers using the definition of Fibonacci numbers.

Solution:
The first five Fibonacci numbers are 1, 1, 2, 3, 5. The sixth Fibonacci is the sum of the two previous Fibonacci numbers. Thus,

$F_6 = F_5 + F_4$
$= 3 + 5$
$= 8$

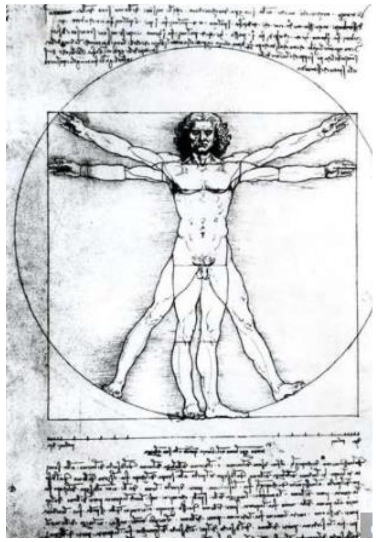
The seventh Fibonacci number is

$F_7 = F_6 + F_5$
$= 5 + 8$
$= 13$

II. Golden Ratio

Golden ratio, also known as the **golden section, golden mean**, or **divine proportion**, in mathematics, the irrational number $(1 + \sqrt{5})/2$, often denoted by the Greek letter ϕ or τ, which is approximately equal to 1.618. It is the ratio of a line segment cut into two pieces of different lengths such that the ratio of the whole segment to that of the longer segment is equal to the ratio of the longer segment to the shorter segment. The origin of this number can be traced back to Euclid, who mentions it as the "extreme and mean ratio" in the Elements. In terms of present day algebra, letting the length of the shorter segment be one unit and the length of the longer segment be x units gives rise to the equation $(x + 1)/x = x/1$; this may be rearranged to form the quadratic equation $x^2 – x – 1 = 0$, for which the positive solution is $x = (1 + \sqrt{5})/2$, the golden ratio.

The ancient Greeks recognized this "dividing" or "sectioning" property, a phrase that was ultimately shortened to simply "the section." It was more than 2,000 years later that both "ratio" and "section" were designated as "golden" by German mathematician Martin Ohm in 1835. The Greeks also had observed that the golden ratio provided the most aesthetically pleasing proportion of sides of a rectangle, a notion that was enhanced during the Renaissance by, for example, the work of the Italian polymath Leonardo da Vinci and the publication of *De divina proportione* (1509; *Divine Proportion*), written by the Italian mathematician Luca Pacioli and illustrated by Leonardo.



The golden ratio occurs in many mathematical contexts. It is geometrically constructible by straightedge and compass, and it occurs in the investigation of the Archimedean and Platonic solids. It is the limit of the ratios of consecutive terms of the Fibonacci number sequence 1, 1, 2, 3, 5, 8, 13,…, in which each term beyond the second is the sum of the previous two, and it is also the value of the most basic of continued fractions, namely 1 + 1/(1 + 1/(1 + 1/(1 +⋯.

Congratulations! You have earned an Independent Reading Badge

Application | Fact or Bluff!

Test your understanding. Each number has 2 icons, color the **fact** icon if the statement is correct, and the **bluff** icon if otherwise. (10 points)

FACT	BLUFF	1. Fibonacci sequence begins with one.
FACT	BLUFF	2. Golden ratio is approximately equal to π or 3.1416.
FACT	BLUFF	3. Each of the subsequent number is the sum of the two preceding numbers
FACT	BLUFF	4. Carl Friedrich Gauss is the inventor of the famous sequence called "The Fibonnacci Sequence".
FACT	BLUFF	5. Leonardo Da Vinci incorporated Fibonacci sequence in his own paintings.

Congratulations! You have earned a Practice Badge

Generalization | Reflect the use of it!

In an essay no more than 100 words, how do you think learning Fibonacci sequence and golden ratio can help you being a future teacher?

____
____
____
____
____
____
____
____

Congratulations! You have earned a Reflection Badge

Assessment: | It is your turn!

Answer the following:

1. Who was the first mathematician defined golden ratio?
2. Who is Fibonacci and what did he do?
3. Give the first 25 terms of the Fibonacci sequence.
4. The golden ratio (shoulder to waist) is the most important ratio for achieving the body proportions like that of a Greek god. Now, measure your shoulder circumference s and then your waist size w. Then divide s by w. Is the result roughly the golden ratio? If not, then what must be your ideal waist size to get the golden ratio.

Congratulations! You have earned a Quizzer Badge

List of References:

Aufmann, R. N. et al. (2013). Mathematical Excursions, 3rd Edition. Cengage Learning.

https://www.britannica.com/science/golden-ratio

Icons and Images

https://www.google.com/search?q=fact+fist&tbm=isch&ved=2ahUKEwj5psnQ45TqAhUnw4sBHVqRCAkQ2-cCegQIABAA&oq=fact+fist&gs_lcp=CgNpbWcQAzoCCAA6BggAEAoQGDoECAAQGDoGCAAQCBAeUJjkBFj6mgVgkpwFaAVwAHgCgAGLA4gBnA-SAQcyLjYuMS4ymAEAoAEBqgELZ3dzLXdpei1pbWc&sclient=img&ei=d03wXrmPNaeGr7wP2qKiSA#imgrc=_nh5wNTYRGcWBM

https://www.freepik.com/free-vector/collection-flat-colored-badge_861042.htm

https://www.freepik.com/premium-vector/gold-medal-golden-1st-place-badge-sport-game-golden-challenge-award-red-ribbon-realistic_6373639.htm

https://www.vectorstock.com/royalty-free-vector/modern-flat-design-badge-icon-vector-10839496

https://learnodo-newtonic.com/fibonacci-facts/fibonacci-numbers-in-petals-of-various-flowers

https://jcdr.net/articles/PDF/13317/42772_PD_(V-2-PK_PrG_OM_SL)_GC(PrG_KM_OM)_PN(SL).pdf

http://2muchfun.info/nautilusshell.html

Congratulations! You have earned Lesson Completer Badge.